Chapter 38  -  Diffraction Patterns and Polarization

P38.2 The positions of the first-order minima are \( \frac{y}{L} = \sin \theta = \pm \frac{\lambda}{a} \). Thus, the spacing between these two minima is \( \Delta y = 2 \left( \frac{\lambda}{a} \right) L \) and the wavelength is
\[
\lambda = \left( \frac{\Delta y}{2} \right) \left( \frac{a}{L} \right) = \left( \frac{4.10 \times 10^{-3} \text{ m}}{2} \right) \left( \frac{0.550 \times 10^{-3} \text{ m}}{2.06 \text{ m}} \right) = 547 \text{ nm}
\]

P38.3
\[
\frac{y}{L} \approx \sin \theta = \frac{m\lambda}{a} \quad \Delta y = 3.00 \times 10^{-3} \text{ nm}
\]
\[
\Delta m = 3 - 1 = 2 \quad \text{and} \quad a = \frac{\Delta m\lambda L}{\Delta y} = \frac{2 \left( 690 \times 10^{-9} \text{ m} \right) \left( 0.500 \text{ m} \right) \left( 3.00 \times 10^{-3} \text{ m} \right)}{547 \text{ nm}} = 2.30 \times 10^{-4} \text{ m}
\]

P38.7 \( \sin \theta = \frac{y}{L} = \frac{4.10 \times 10^{-3} \text{ m}}{1.20 \text{ m}} \)

We define \( \phi = \frac{\pi \sin \theta}{\lambda} = \frac{\pi \left( 4.00 \times 10^{-4} \text{ m} \right) \left( 4.10 \times 10^{-3} \text{ m} \right)}{546.1 \times 10^{-9} \text{ m} \left( 1.20 \text{ m} \right)} = 7.86 \text{ rad} \)

\[
\frac{I}{I_{\max}} = \left[ \frac{\sin \left( \frac{\phi}{2} \right)}{\phi} \right]^2 = \left[ \frac{\sin \left( \frac{7.86}{2} \right)}{7.86} \right]^2 = 1.62 \times 10^{-2}
\]

P38.13 Undergoing diffraction from a circular opening, the beam spreads into a cone of half-angle \( \theta_{\min} = 1.22 \frac{\lambda}{D} = 1.22 \left( \frac{632.8 \times 10^{-9} \text{ m}}{0.00500 \text{ m}} \right) = 1.54 \times 10^{-4} \text{ rad} \)

The radius of the beam ten kilometers away is, from the definition of radian measure,
\[
r_{\text{beam}} = \theta_{\min} \left( 1.00 \times 10^4 \text{ m} \right) = 1.544 \text{ m}
\]

and its diameter is \( d_{\text{beam}} = 2r_{\text{beam}} = 3.09 \text{ m} \).

P38.18 \( 1.22 \frac{\lambda}{D} = \frac{d}{L} \) \quad \frac{\lambda}{f} = 0.0200 \text{ m}

\[
D = 2.10 \text{ m} \quad L = 9 \text{ km} \quad d = 1.22 \left( \frac{0.0200 \text{ m}}{2.10 \text{ m}} \right) \left( 9 \text{ km} \right) = 105 \text{ m}
\]
The principal maxima are defined by

\[ dsin \theta = m\lambda \quad m = 0, 1, 2, \ldots \]

For \( m = 1 \),

\[ \lambda = dsin \theta \]

where \( \theta \) is the angle between the central \((m=0)\) and the first order \((m=1)\) maxima. The value of \( \theta \) can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

\[ \tan \theta = \frac{0.488 \text{ m}}{1.72 \text{ m}} = 0.284 \]

so \( \theta = 15.8^\circ \)

and \( sin \theta = 0.273 \)

The distance between grating “slits” equals the reciprocal of the number of grating lines per centimeter

\[ d = \frac{1}{5310 \text{ cm}^{-1}} = 1.88 \times 10^{-4} \text{ cm} = 1.88 \times 10^3 \text{ nm} \]

The wavelength is \( \lambda = dsin \theta = (1.88 \times 10^3 \text{ nm})(0.273) = 514 \text{ nm} \)

\[ \sin \theta = 0.350 : \quad d = \frac{\lambda}{\sin \theta} = \frac{632.8 \text{ nm}}{0.350} = 1.81 \times 10^3 \text{ nm} \]

Line spacing = \[ 1.81 \mu \text{m} \]

\[ d = \frac{1.00 \times 10^{-3} \text{ m/mm}}{250 \text{ lines/mm}} = 4.00 \times 10^{-6} \text{ m} = 4000 \text{ nm} \]

\[ dsin \theta = m\lambda \Rightarrow m = \frac{dsin \theta}{\lambda} \]

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

\[ m_{\text{max}} = \frac{dsin \theta_{\text{max}}}{\lambda} = \frac{(4000 \text{ nm}) \sin 90.0^\circ}{700 \text{ nm}} = 5.71 \]

or \boxed{5 \text{ orders is the maximum}}

(b) The highest order in which the violet end of the spectrum can be seen is:

\[ m_{\text{max}} = \frac{dsin \theta_{\text{max}}}{\lambda} = \frac{(4000 \text{ nm}) \sin 90.0^\circ}{400 \text{ nm}} = 10.0 \]

or \boxed{10 \text{ orders in the short-wavelength region}}
\[ d = \frac{1}{4 \times 200/\text{cm}} = 2.38 \times 10^{-6} \text{ m} = 2380 \text{ nm} \]

\[ d \sin \theta = m \lambda \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{m \lambda}{d}\right) \quad \text{and} \]

\[ y = L \tan \theta = L \tan \left[ \sin^{-1}\left(\frac{m \lambda}{d}\right) \right] \]

Thus,

\[ \Delta y = L \left\{ \tan \left[ \sin^{-1}\left(\frac{m_2 \lambda}{d}\right) \right] - \tan \left[ \sin^{-1}\left(\frac{m_1 \lambda}{d}\right) \right] \right\} \]

For \( m = 1 \),

\[ \Delta y = (2.00 \text{ m}) \left\{ \tan \left[ \sin^{-1}\left(\frac{589.6}{2380}\right) \right] - \tan \left[ \sin^{-1}\left(\frac{589}{2380}\right) \right] \right\} = 0.554 \text{ mm} \]

For \( m = 2 \),

\[ \Delta y = (2.00 \text{ m}) \left\{ \tan \left[ \sin^{-1}\left(\frac{2 \times 589.6}{2380}\right) \right] - \tan \left[ \sin^{-1}\left(\frac{2 \times 589}{2380}\right) \right] \right\} = 1.54 \text{ mm} \]

For \( m = 3 \),

\[ \Delta y = (2.00 \text{ m}) \left\{ \tan \left[ \sin^{-1}\left(\frac{3 \times 589.6}{2380}\right) \right] - \tan \left[ \sin^{-1}\left(\frac{3 \times 589}{2380}\right) \right] \right\} = 5.04 \text{ mm} \]

Thus, the observed order must be \( m = 2 \).

\[ \lambda = \frac{2dsin \theta}{m} = \frac{2 \left(0.353 \times 10^{-9} \text{ m}\right) \sin 7.60^\circ}{1} = 9.34 \times 10^{-11} \text{ m} = 0.0934 \text{ nm} \]

\[ I = I_{\text{max}} \cos^2 \theta \quad \Rightarrow \quad \theta = \cos^{-1} \sqrt{\frac{I}{I_{\text{max}}}} \]

(a) \[ \frac{I}{I_{\text{max}}} = \frac{1}{3.00} \quad \Rightarrow \quad \theta = \cos^{-1} \sqrt{\frac{1}{3.00}} = 54.7^\circ \]

(b) \[ \frac{I}{I_{\text{max}}} = \frac{1}{5.00} \quad \Rightarrow \quad \theta = \cos^{-1} \sqrt{\frac{1}{5.00}} = 63.4^\circ \]

(c) \[ \frac{I}{I_{\text{max}}} = \frac{1}{10.0} \quad \Rightarrow \quad \theta = \cos^{-1} \sqrt{\frac{1}{10.0}} = 71.6^\circ \]
**P38.36** By Brewster’s law, \[ n = \tan \theta_p = \tan(48.0^\circ) = 1.11 \]

**P38.41** (a) Let \( I_0 \) represent the intensity of unpolarized light incident on the first polarizer. In Malus's law the average value of the cosine-squared function is 1/2, so the first filter lets through 1/2 of the incident intensity. Of the light reaching them, the second filter passes \( \cos^2 45^\circ = 1/2 \) and the third filter also \( \cos^2 45^\circ = 1/2 \). The transmitted intensity is then \( I_0(1/2)(1/2)(1/2) = 0.125 I_0 \). The reduction in intensity is by a factor of \( 0.875 \) of the incident intensity.

(b) By the same logic as in part (a) we have transmitted \( I_0(1/2)(\cos^2 30^\circ)(\cos^2 30^\circ)(\cos^2 30^\circ) = I_0(1/2)(\cos^2 30^\circ)^3 = 0.211 I_0 \). Then the fraction absorbed is \( 0.789 \).

(c) Yet again we compute transmission \( I_0(1/2)(\cos^2 15^\circ)^6 = 0.330 I_0 \). And the fraction absorbed is \( 0.670 \).

(d) We can get more and more of the incident light through the stack of ideal filters, approaching 50%, by reducing the angle between the transmission axes of each one and the next.

**P38.49** \( d = \frac{1}{400 \text{ mm}^{-1}} = 2.50 \times 10^{-6} \text{ m} \)

(a) \( ds \sin \theta = m \lambda \)

(b) \( \lambda = \frac{541 \times 10^{-9} \text{ m}}{1.33} = 4.07 \times 10^{-7} \text{ m} \)

(c) \( ds \sin \theta_a = 2 \lambda \) and \( ds \sin \theta_b = \frac{2 \lambda}{n} \)

combine by substitution to give \( n \sin \theta_b = (1) \sin \theta_a \)

**P38.50** (a) \( \lambda = \frac{v}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.40 \times 10^9 \text{ s}^{-1}} = 0.214 \text{ m} \)

(b) \( \theta_{min} = 1.22 \frac{\lambda}{D} : \) \( \theta_{min} = 1.22 \left( \frac{0.214 \text{ m}}{3.60 \times 10^4 \text{ m}} \right) = 7.26 \mu \text{rad} \)

(c) \( \theta_{min} = 7.26 \mu \text{rad} \left( \frac{180 \times 60 \times 60 \text{ s}}{\pi} \right) = 1.50 \text{ arc seconds} \)

(b) \( \theta_{min} = \frac{d}{L} : \) \( d = \theta_{min} L = \left( 7.26 \times 10^{-6} \text{ rad} \right) \left( 26,000 \text{ ly} \right) = 0.189 \text{ ly} \)

(c) It is not true for humans, but we assume the hawk’s visual acuity is limited only by Rayleigh's criterion.
\[
\theta_{\text{min}} = 1.22 \frac{\lambda}{D} \quad \theta_{\text{min}} = 1.22 \left( \frac{500 \times 10^{-9} \text{ m}}{12.0 \times 10^{-3} \text{ m}} \right) = 50.8 \mu \text{rad} \quad (10.5 \text{ seconds of arc})
\]

(d) \[d = \theta_{\text{min}} \times \frac{L}{90} = \left( 50.8 \times 10^{-6} \text{ rad} \right) \times (30.0 \text{ m}) = 1.52 \times 10^{-3} \text{ m} = 1.52 \text{ mm}\]

*P38.54* (a) For the air-to-water interface,
\[
\tan \theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = 1.33 \quad \text{and} \quad \tan \theta = \frac{n_{\text{slab}}}{n_{\text{water}}} = \frac{n}{1.33}
\]
and \[(1.00)\sin \theta_p = (1.33)\sin \theta_2 \]
\[
\theta_2 = \sin^{-1} \left( \frac{\sin 53.1^{\circ}}{1.33} \right) = 36.9^{\circ}
\]
For the water-to-slab interface,
\[
\tan \theta_p = \tan \theta_3 = \frac{n_{\text{slab}}}{n_{\text{water}}} = \frac{n}{1.33}
\]
so \[\theta_3 = \tan^{-1} \left( \frac{n}{1.33} \right) \]

The angle between surfaces is \[\theta = \theta_3 - \theta_2 = \tan^{-1} \left( \frac{n}{1.33} \right) - 36.9^{\circ}\].

(b) If we imagine \(n \to \infty\), then \(\theta \to 53.1^{\circ}\). The material of the slab in this case is higher in optical density than any gem. The light in the water skims along the upper surface of the slab.

(c) If we imagine \(n = 1\), then \(\theta = 0\). The slab is so low in optical density that it is like air. The light strikes parallel surfaces as it enters and exits the water, both at the polarizing angle.

P38.57 For the limiting angle of resolution between lines we assume
\[
\theta_{\text{min}} = 1.22 \frac{\lambda}{D} = 1.22 \left( \frac{550 \times 10^{-9} \text{ m}}{5.00 \times 10^{-3} \text{ m}} \right) = 1.34 \times 10^{-4} \text{ rad}.
\]
Assuming a picture screen with vertical dimension \(\ell\), the minimum viewing distance for no visible lines is found from \(\theta_{\text{min}} = \frac{\ell}{485L}\). The desired ratio is then
\[
\frac{L}{\ell} = \frac{1}{485\theta_{\text{min}}} = \frac{1}{485 \left( 1.34 \times 10^{-4} \text{ rad} \right)} = 15.4
\]

When the pupil of a human eye is wide open, its actual resolving power is significantly poorer than Rayleigh's criterion suggests.

P38.59 (a) From Equation 38.1,
\[
\theta = \sin^{-1} \left( \frac{m\lambda}{a} \right)
\]
In this case \(m = 1\) and \[
\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8} \text{ m/s}}{7.50 \times 10^{9} \text{ Hz}} = 4.00 \times 10^{-2} \text{ m}
\]
Thus, \[ \theta = \sin^{-1}\left(\frac{4.00 \times 10^{-2} \text{ m}}{6.00 \times 10^{-2} \text{ m}}\right) = 41.8^\circ \]

(b) From Equation 38.2, \[ \frac{l}{l_{\text{max}}} = \left[\frac{\sin(\phi)}{\phi}\right]^2 \] where \( \phi = \frac{\pi \sin \theta}{\lambda} \)

When \( \theta = 15.0^\circ \), \[ \phi = \frac{\pi (0.060 \text{ m}) \sin 15.0^\circ}{0.040 \text{ m}} = 1.22 \text{ rad} \]

and \[ \frac{l}{l_{\text{max}}} = \left[\frac{\sin(1.22 \text{ rad})}{1.22 \text{ rad}}\right]^2 = 0.593 \]

(c) \( \sin \theta = \frac{\lambda}{a} \) so \( \theta = 41.8^\circ \):

This is the minimum angle subtended by the two sources at the slit. Let \( \alpha \) be the half angle between the sources, each a distance \( \ell = 0.100 \text{ m} \) from the center line and a distance \( L \) from the slit plane. Then,
\[
L = \ell \cot \alpha = (0.100 \text{ m}) \cot\left(\frac{41.8^\circ}{2}\right) = 0.262 \text{ m}
\]

*P38.60* (a) The first sheet transmits one-half the intensity of the originally unpolarized light, because the average value of the cosine-squared function in Malus’s law is one-half. Then
\[ \frac{l}{l_{\text{max}}} = \frac{1}{2} (\cos^2 45.0^\circ) (\cos^2 45.0^\circ) = \frac{1}{8} \]

(b) No recipes remain. The two experiments follow precisely analogous steps, but the results are different. The middle filter in part (a) changes the polarization state of the light that passes through it, but the recipe selections do not change individual recipes. The result for light gives us a glimpse of how quantum-mechanical measurements differ from classical measurements.

P38.62 (a) The angles of bright beams diffracted from the grating are given by \( (d) \sin \theta = m\lambda \). The angular dispersion is defined as the derivative \( \frac{d\theta}{d\lambda} \):
\[
(d) \cos \theta \frac{d\theta}{d\lambda} = m
\]
\[
\frac{d\theta}{d\lambda} = \frac{m}{d\cos \theta}
\]

(b) For the average wavelength 578 nm,
\[
d\sin \theta = m\lambda \quad \frac{0.02 \text{ m}}{8000} \sin \theta = 2\left(578 \times 10^{-9} \text{ m}\right)
\]
\[
\theta = \sin^{-1}\left(\frac{2 \times 578 \times 10^{-9} \text{ m}}{2.5 \times 10^{-6} \text{ m}}\right) = 27.5^\circ
\]
The separation angle between the lines is

\[
\Delta \theta = \frac{d\theta}{d\lambda} \Delta \lambda = \frac{m}{d \cos \theta} \Delta \lambda = \frac{2}{2.5 \times 10^{-6} \text{ m} \cos 27.5^\circ} \times 2.11 \times 10^{-9} \text{ m}
\]

\[
= 0.00190 = 0.00190 \text{ rad} = 0.00190 \text{ rad} \left( \frac{180^\circ}{\pi \text{ rad}} \right) = 0.109^\circ
\]

P38.63 (a) The E and O rays, in phase at the surface of the plate, will have a phase difference

\[
\theta = \left( \frac{2\pi}{\lambda} \right) \delta
\]

after traveling distance \(d\) through the plate. Here \(\delta\) is the difference in the \textit{optical path} lengths of these rays. The optical path length between two points is the product of the actual path length \(d\) and the index of refraction. Therefore,

\[
\delta = |n_O - n_E|
\]

The absolute value is used since \(\frac{n_O}{n_E}\) may be more or less than unity. Therefore,

\[
\theta = \left( \frac{2\pi}{\lambda} \right) |n_O - n_E| = \left( \frac{2\pi}{\lambda} \right) d |n_O - n_E|
\]

(b) \[
\frac{\lambda \theta}{2\pi |n_O - n_E|} = \frac{(550 \times 10^{-9} \text{ m})(\pi/2)}{2\pi |1.544 - 1.553|} = 1.53 \times 10^{-5} \text{ m} = 15.3 \mu \text{m}
\]