Chapter 37 - Interference of Light Waves

P37.2 $y_{bright} = \frac{\lambda L}{d}$

For $m = 1$, 
$$\lambda = \frac{yd}{L} = \frac{\left(3.40 \times 10^{-3} \text{ m}\right)\left(5.00 \times 10^{-4} \text{ m}\right)}{3.30 \text{ m}} = 515 \text{ nm}$$

P37.5 In the equation $d\sin \theta = \left(m + \frac{1}{2}\right)\lambda$

The first minimum is described by $m = 0$

and the tenth by $m = 9$:

$$\sin \theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$$

Also,

$$\tan \theta = \frac{y}{L}$$

but for small $\theta$,

$$\sin \theta \approx \tan \theta$$

Thus,

$$d = \frac{9.5\lambda}{\sin \theta} = \frac{9.5\lambda}{y}$$

$$d = \frac{9.5\left(5.890 \times 10^{-10} \text{ m}\right)\left(2.00 \text{ m}\right)}{7.26 \times 10^{-3} \text{ m}} = 1.54 \times 10^{-3} \text{ m} = \boxed{1.54 \text{ mm}}$$

P37.7 (a) For the bright fringe,

$$y_{bright} = \frac{mL}{d} \quad \text{where} \quad m = 1$$

$$y = \left(\frac{546.1 \times 10^{-9} \text{ m}}{0.250 \times 10^{-3} \text{ m}}\right)\left(1.20 \text{ m}\right) = 2.62 \times 10^{-3} \text{ m} = \boxed{2.62 \text{ mm}}$$

(b) For the dark bands, $y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right); \quad m = 0, 1, 2, 3, \ldots$

$$y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1 + \frac{1}{2}\right) - \left(0 + \frac{1}{2}\right)\right] = \frac{\lambda L}{d}(1)$$

$$= \frac{\left(546.1 \times 10^{-9} \text{ m}\right)\left(1.20 \text{ m}\right)}{0.250 \times 10^{-3} \text{ m}}$$

$$\Delta y = \boxed{2.62 \text{ mm}}$$

*P37.15 (a) The path difference $\delta = d\sin \theta$ and when $L >> y$

$$\delta = \frac{yd}{L} = \frac{\left(1.80 \times 10^{-2} \text{ m}\right)\left(1.50 \times 10^{-4} \text{ m}\right)}{1.40 \text{ m}} = 1.93 \times 10^{-6} \text{ m} = \boxed{1.93 \mu m}$$
(b) \[
\frac{\delta}{\lambda} = \frac{1.93 \times 10^{-6} \text{ m}}{6.43 \times 10^{-7} \text{ m}} = 3.00, \text{ or } \delta = 3.00\lambda
\]

(c) Point P will be a maximum because the path difference is an integer multiple of the wavelength.

\[P37.17 \quad I_{av} = I_{max} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right)\]

For small \(\theta\), \(\sin \theta = \frac{y}{L}\)

and \(I_{av} = 0.750I_{max}\)

\[y = \frac{\lambda L}{\pi d} \cos^{-1} \left( \frac{I_{av}}{I_{max}} \right)\]

\[y = \frac{(6.00 \times 10^{-7}) (1.20 \text{ m}) \cos^{-1} \left( \frac{0.750I_{max}}{I_{max}} \right)}{\pi (2.50 \times 10^{-3} \text{ m})} = 48.0 \mu\text{m}\]

\[P37.23 \quad (a) \]

The light reflected from the top of the oil film undergoes phase reversal. Since \(1.45 > 1.33\), the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have

\[2nt = \left( m + \frac{1}{2} \right) \frac{\lambda}{2}\]

or \(\lambda_m = \frac{2nt}{m + (1/2)} = \frac{2(1.45)(280 \text{ nm})}{m + (1/2)}\)

Substituting for \(m\) gives:

\(m = 0, \quad \lambda_0 = 1620 \text{ nm} \) (infrared)

\(m = 1, \quad \lambda_1 = 541 \text{ nm} \) (green)

\(m = 2, \quad \lambda_2 = 325 \text{ nm} \) (ultraviolet)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in reflected light is green.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

\[2nt = m\lambda\]

or \(\lambda_m = \frac{2nt}{m} = \frac{812 \text{ nm}}{m}\)

Substituting for \(m\) gives:

\(m = 1, \quad \lambda_1 = 812 \text{ nm} \) (near infrared)

\(m = 2, \quad \lambda_2 = 406 \text{ nm} \) (violet)

\(m = 3, \quad \lambda_3 = 271 \text{ nm} \) (ultraviolet)
Of these, the only wavelength visible to the human eye (and hence the dominant wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is \textit{violet}.

**P37.27** \[2nt = \left( m + \frac{1}{2} \right) \lambda \quad \text{so} \quad t = \left( m + \frac{1}{2} \right) \frac{\lambda}{2n} \]

Minimum \[t = \left( \frac{1}{2} \right) \left( \frac{500 \text{ nm}}{2(1.30)} \right) = 96.2 \text{ nm} \]

**P37.28** Since the light undergoes a 180° phase change at each surface of the film, the condition for constructive interference is 
\[2nt = m \lambda, \quad \text{or} \quad \lambda = \frac{2nt}{m}. \]
The film thickness is 
\[t = 1.00 \times 10^{-5} \text{ cm} = 1.00 \times 10^{-7} \text{ m} = 100 \text{ nm}. \]
Therefore, the wavelengths intensified in the reflected light are 
\[\lambda = \frac{2(1.38)(100 \text{ nm})}{m} = \frac{276 \text{ nm}}{m} \]
where \(m = 1, 2, 3, \ldots\)
or \(\lambda_1 = 276 \text{ nm}, \lambda_2 = 138 \text{ nm}, \ldots\) All reflection maxima are in the ultraviolet and beyond. \textbf{No visible wavelengths are intensified.}

**P37.31** For destructive interference in the air,
\[2t = m \lambda \]
For 30 dark fringes, including the one where the plates meet,
\[t = \frac{29(600 \text{ nm})}{2} = 8.70 \times 10^{-6} \text{ m} \]
Therefore, the radius of the wire is
\[r = \frac{t}{2} = \frac{8.70 \mu \text{m}}{2} = 4.35 \mu \text{m} \]

**P37.34** Distance \[= 2\left(3.82 \times 10^{-4} \text{ m}\right) = 1.700 \lambda \]
\[\lambda = 4.49 \times 10^{-7} \text{ m} = 449 \text{ nm} \]
The light is \textit{blue}.

**P37.35** When the mirror on one arm is displaced by \(\Delta \ell\), the path difference changes by \(2\Delta \ell\).
A shift resulting in the reversal between dark and bright fringes requires a path length change of one-half wavelength. Therefore, \(2\Delta \ell = \frac{m\lambda}{2}\), where in this case, \(m = 250\).
\[ \Delta \ell = m \frac{\lambda}{4} = \frac{(250)(6.328 \times 10^{-7} \text{ m})}{4} = 39.6 \mu \text{m} \]

**P37.36**  
Counting light going both directions, the number of wavelengths originally in the cylinder is \( m_1 = \frac{2L}{\lambda} \). It changes to \( m_2 = \frac{2L}{\lambda/n} = \frac{2nL}{\lambda} \) as the cylinder is filled with gas. If \( N \) is the number of bright fringes passing, \( N = m_2 - m_1 = \frac{2L}{\lambda}(n-1) \), or the index of refraction of the gas is
\[ n = 1 + \frac{N \lambda}{2L} \]

**P37.44**  
If the center point on the screen is to be a dark spot rather than bright, passage through the plastic must delay the light by one-half wavelength. Calling the thickness of the plastic \( t \),
\[ \frac{t + 1}{\frac{\lambda}{2}} = \frac{nt}{\frac{\lambda}{n}} \quad \text{or} \quad t = \frac{\lambda}{2(n-1)} \]
where \( n \) is the index of refraction for the plastic.

**P37.45**  
No phase shift upon reflection from the upper surface (glass to air) of the film, but there will be a shift of \( \frac{\lambda}{2} \) due to the reflection at the lower surface of the film (air to metal). The total phase difference in the two reflected beams is then
\[ \delta = 2nt + \frac{\lambda}{2} \]
For constructive interference, \( \delta = m\lambda \)
or
\[ 2(1.00)t + \frac{\lambda}{2} = m\lambda \]
Thus, the film thickness for the \( m \)th order bright fringe is
\[ t_m = \left( m - \frac{1}{2} \right) \frac{\lambda}{2} = m\left( \frac{\lambda}{2} \right) - \frac{\lambda}{4} \]
and the thickness for the \( m-1 \) bright fringe is:
\[ t_{m-1} = (m-1)\left( \frac{\lambda}{2} \right) - \frac{\lambda}{4} \]
Therefore, the change in thickness required to go from one bright fringe to the next is
\[ \Delta t = t_m - t_{m-1} = \frac{\lambda}{2} \]
To go through 200 bright fringes, the change in thickness of the air film must be:
\[ 200\left( \frac{\lambda}{2} \right) = 100\lambda \]
Thus, the increase in the length of the rod is
\[ \Delta L = 100\lambda = 100 \left( 5.00 \times 10^{-7} \text{ m} \right) = 5.00 \times 10^{-5} \text{ m} \]

From \[ \Delta L = L/\alpha \Delta T \]
we have: \[ \alpha = \frac{\Delta L}{L/\Delta T} = \frac{5.00 \times 10^{-5} \text{ m}}{(0.100 \text{ m})(25.0^\circ \text{C})} = 2.00 \times 10^{-6} \text{ } ^\circ \text{C}^{-1} \]

**P37.48** For destructive interference, the path length must differ by \( m\lambda \). We may treat this problem as a double slit experiment if we remember the light undergoes a \( \frac{\pi}{2} \)-phase shift at the mirror. The second slit is the mirror image of the source, 1.00 cm below the mirror plane. Modifying Equation 37.7,

\[ y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1 \left( 5.00 \times 10^{-7} \text{ m} \right) \left( 100 \text{ m} \right)}{(2.00 \times 10^{-2} \text{ m})} = 2.50 \text{ mm} \]

**P37.49** \[ 2\sqrt{(15.0 \text{ km})^2 + h^2} = 30.175 \text{ km} \]

\[ (15.0 \text{ km})^2 + h^2 = 227.63 \]

\[ h = 1.62 \text{ km} \]

**FIG. P37.49**

**P37.51** From Equation 37.14,

\[ \frac{l}{I_{\text{max}}} = \cos^2 \left( \frac{\pi y d}{\lambda L} \right) \]

Let \( \lambda_2 \) equal the wavelength for which

\[ \frac{l}{I_{\text{max}}} \rightarrow \frac{l}{I_{\text{max}}} = 0.640 \]

Then

\[ \lambda_2 = \frac{\pi y d/L}{\cos^{-1} \left( \frac{l}{I_{\text{max}}} \right)^{1/2}} \]

But

\[ \frac{\pi y d}{L} = \lambda_1 \cos^{-1} \left( \frac{l}{I_{\text{max}}} \right)^{1/2} = (600 \text{ nm}) \cos^{-1} (0.900) = 271 \text{ nm} \]

Substituting this value into the expression for \( \lambda_2 \),

\[ \lambda_2 = \frac{271 \text{ nm}}{\cos^{-1} (0.640)^{1/2}} = 421 \text{ nm} \]

Note that in this problem, \( \cos^{-1} \left( \frac{l}{I_{\text{max}}} \right)^{1/2} \) must be expressed in radians.

**P37.57** Call \( t \) the thickness of the film. The central maximum corresponds to zero phase difference. Thus, the added distance \( \Delta r \) traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. The phase difference \( \phi \) is

\[ \phi = 2\pi \left( \frac{t}{\lambda_2} \right) (n-1) \]
The corresponding difference in path length $\Delta r$ is

$$\Delta r = \phi \left( \frac{2a}{\lambda_a} \right) = 2\pi \left( \frac{t}{\lambda_a} \right) (n-1) \left( \frac{2a}{2\pi} \right) = t(n-1)$$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle $\theta$ may be expressed as

$$\tan \theta = \frac{\Delta r}{d} = \frac{y'}{L}$$

Eliminating $\Delta r$ by substitution,

$$\frac{y'}{L} = \frac{t(n-1)}{d}$$

**P37.59**

(a) Constructive interference in the reflected light requires $2t = \left( m + \frac{1}{2} \right) \lambda$. The first bright ring has $m = 0$ and the 55th has $m = 54$, so at the edge of the lens

$$t = \frac{54.5 \times 650 \times 10^{-9} \text{ m}}{2} = 17.7 \mu\text{m}$$

Now from the geometry in textbook Figure 37.12, we can find the distance $t$ from the curved surface down to the flat plate by considering distances down from the center of curvature:

$$\sqrt{R^2 - r^2} = R - t \quad \text{or} \quad R^2 - r^2 = R^2 - 2Rt + t^2$$

$$R = \frac{r^2 + t^2}{2t} = \frac{(5.00 \times 10^{-2} \text{ m})^2 + (1.77 \times 10^{-5} \text{ m})^2}{2(1.77 \times 10^{-5} \text{ m})} = 70.6 \text{ m}$$

(b) $\frac{1}{f} = (n-1) \left( \frac{1 - 1}{R_2 - R_2} \right) = 0.520 \left( \frac{1}{\infty} - \frac{1}{70.6 \text{ m}} \right) \quad \text{so} \quad f = 136 \text{ m}$

**P37.60**

The shift between the waves reflecting from the top and bottom surfaces of the film at the point where the film has thickness $t$ is

$$\delta = 2tn_{film} + \frac{\lambda}{2}$$

with the factor of $\frac{\lambda}{2}$ being due to a phase reversal at one of the surfaces.

For the dark rings (destructive interference), the total shift should be $\delta = \left( m + \frac{1}{2} \right) \lambda$ with $m = 0, 1, 2, 3, \ldots$. This requires that

$$t = \frac{m\lambda}{2n_{film}}.$$

To find $t$ in terms of $r$ and $R$,

$$R^2 = r^2 + (R-t)^2 \quad \text{so} \quad r^2 = 2Rt + t^2$$

Since $t$ is much smaller than $R$,

$$t^2 \ll 2Rt \quad \text{and} \quad r^2 \approx 2Rt = 2R \left( \frac{m\lambda}{2n_{film}} \right)$$
Thus, where \( m \) is an integer,

\[
r \approx \frac{m \lambda R}{n \text{film}}
\]

P37.63

(a) Bright bands are observed when

\[
2nt = \left( m + \frac{1}{2} \right) \lambda.
\]

Hence, the first bright band \( (m = 0) \) corresponds to

\[
nt = \frac{\lambda}{4}.
\]

Since \( \frac{x_1}{x_2} = \frac{t_1}{t_2} \),

we have

\[
x_2 = x_1 \left( \frac{t_2}{t_1} \right) = x_1 \left( \frac{\lambda_2/\lambda_1}{1} \right) = (3.00 \text{ cm}) \left( \frac{680 \text{ nm}}{420 \text{ nm}} \right) = 4.86 \text{ cm}
\]

(b) \( t_1 = \frac{\lambda_1}{4n} - \frac{420 \text{ nm}}{4(1.33)} = 78.9 \text{ nm} \)

\( t_2 = \frac{\lambda_2}{4n} = \frac{680 \text{ nm}}{4(1.33)} = 128 \text{ nm} \)

(c) \( \theta \approx \tan \theta = \frac{t_1}{x_1} = \frac{78.9 \text{ nm}}{3.00 \text{ cm}} = 2.63 \times 10^{-6} \text{ rad} \)

P37.66

Superposing the two vectors, \( E_R = \left| \vec{E}_1 + \vec{E}_2 \right| \)

\[
E_R \left( \vec{E}_1 + \vec{E}_2 \right) = \sqrt{\left( E_0 + \frac{E_0^2}{3} \cos \phi \right)^2 + \left( E_0 + \frac{E_0^2}{3} \sin \phi \right)^2} = \sqrt{E_0^2 + \frac{2}{3} E_0^2 \cos \phi + \frac{E_0^2}{9} \cos^2 \phi + \frac{E_0^2}{9} \sin^2 \phi}
\]

\[E_R = \sqrt{\frac{10}{9} E_0^2 + \frac{2}{3} E_0^2 \cos \phi}\]

Since intensity is proportional to the square of the amplitude,

\[I = \frac{10}{9} I_{max} + \frac{2}{3} I_{max} \cos \phi\]

Using the trigonometric identity \( \cos \phi = 2 \cos^2 \phi/2 - 1 \), this becomes

\[I = \frac{10}{9} I_{max} + \frac{2}{3} I_{max} \left( 2 \cos^2 \phi/2 - 1 \right) = \frac{4}{9} I_{max} + \frac{4}{3} I_{max} \cos^2 \phi/2\]

or

\[I = \frac{4}{9} I_{max} \left( 1 + 3 \cos^2 \phi \right)\]