The flatness of the mirror is described by 

\[ R = \infty, \quad f = \infty \]

and 

\[ \frac{1}{f} = 0 \]

By our general mirror equation,

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = 0 \]

or 

\[ q = -p \]

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

\[ M = \frac{-q}{p} = \frac{h'}{h} \]

so 

\[ h' = h = 70.0 \text{ inches} \]

The required height of the mirror is defined by the triangle from the person’s eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

\[ h' \left( \frac{p}{p-q} \right) = h \left( \frac{p}{2p} \right) = \frac{h'}{2} \]

Thus, the mirror must be at least 35.0 inches high.

(a) 

\[ \frac{1}{p} + \frac{1}{q} = \frac{2}{R} \]

becomes

\[ \frac{1}{q} = \frac{2}{60.0 \text{ cm}} - \frac{1}{90.0 \text{ cm}} \]

\[ q = 45.0 \text{ cm} \]

and

\[ M = \frac{-q}{p} = \frac{-45.0 \text{ cm}}{90.0 \text{ cm}} = -0.500 \]

(b) 

\[ \frac{1}{p} + \frac{1}{q} = \frac{2}{R} \]

becomes

\[ \frac{1}{q} = \frac{2}{60.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} \]

\[ q = -60.0 \text{ cm} \]

and

\[ M = \frac{-q}{p} = \frac{(-60.0 \text{ cm})}{(20.0 \text{ cm})} = 3.00 \]

(c) The image (a) is real, inverted, and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figures 36.13(a) and 36.13(b) in the text, respectively.

(a) 

\[ M = \frac{-q}{p} \]

\[ q - p = 0.60 \text{ m} = 4p - p \]

\[ \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.2 \text{ m}} + \frac{1}{0.8 \text{ m}} \]

\[ f = 160 \text{ mm} \]
(b) \[ M = \frac{1}{2} = \frac{q}{p} \quad \Rightarrow \quad p = -2q \]

\[ |q + p| = 0.20 \text{ m} = -q + p = -q - 2q \]

\[ q = -66.7 \text{ mm} \quad \Rightarrow \quad p = 133 \text{ mm} \]

\[ \frac{1}{p} + \frac{1}{q} + \frac{2}{R} = \frac{1}{0.133 \text{ m}} + \frac{1}{-0.0667 \text{ m}} \]

\[ R = -267 \text{ mm} \]

\[ q = (p + 5.00 \text{ m}) \quad \text{and, since the image must be real,} \]

\[ M = -\frac{q}{p} = 5 \quad \text{or} \quad q = 5p \]

Therefore, \( p + 5.00 \text{ m} = 5p \)

or \( p = 1.25 \text{ m} \quad \text{and} \quad q = 6.25 \text{ m} \)

From \( \frac{1}{p} + \frac{1}{q} = \frac{2}{R} \),

\[ R = \frac{2pq}{p + q} = \frac{2(1.25)(6.25)}{1.25 + 6.25} = 2.08 \text{ m (concave)} \]

(b) From part (a), \( p = 1.25 \text{ m} \); the mirror should be \( 1.25 \text{ m} \) in front of the object.

\[ \text{P36.17} \]

(a) The image starts from a point whose height above the mirror vertex is given by

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \quad \Rightarrow \quad \frac{1}{3.00 \text{ m}} + \frac{1}{0.500 \text{ m}} = \frac{1}{0.600 \text{ m}} \]

Therefore,

\[ q = 0.600 \text{ m} \]

As the ball falls, \( p \) decreases and \( q \) increases. Ball and image pass when \( q_t = p_t \).

When this is true,

\[ \frac{1}{p_t} + \frac{1}{p_t} = \frac{1}{0.500 \text{ m}} = \frac{2}{p_t} \quad \text{or} \quad p_t = 1.00 \text{ m} \]

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when \( p_2 = q_2 = 0 \).

(b) The falling ball passes its real image when it has fallen

\[ 3.00 \text{ m} - 1.00 \text{ m} = 2.00 \text{ m} = \frac{1}{2}gt^2, \quad \text{or when} \quad t = \sqrt{\frac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.639 \text{ s} \]

The ball reaches its virtual image when it has traversed

\[ 3.00 \text{ m} - 0 \text{ m} = 3.00 \text{ m} = \frac{1}{2}gt^2, \quad \text{or at} \quad t = \sqrt{\frac{2(3.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.782 \text{ s} \]
P36.23 From Equation 36.8 \[
\frac{n_1 + n_2}{\rho} = \frac{n_2 - n_1}{q} \frac{1}{R}
\]
Solve for \(q\) to find
\[
q = \frac{n_2 R \rho}{\rho (n_2 - n_1) - n_1 R}
\]
In this case, \(n_1 = 1.50\), \(n_2 = 1.00\), \(R = -15.0\) cm
and \(\rho = 10.0\) cm
So
\[
q = \frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00 - 1.50) - (1.50)(-15.0\text{ cm})} = -8.57\text{ cm}
\]
Therefore, the apparent depth is 8.57 cm

*P36.25 (a) The center of curvature is on the object side, so the radius of curvature is negative.
\[
\frac{n_1 + n_2}{\rho} = \frac{n_2 - n_1}{q} \frac{1}{R}
\]
becomes
\[
\frac{1.33}{30\text{ cm}} + \frac{1.00}{q} = \frac{1.00 - 1.33}{-80\text{ cm}} \quad q = -24.9\text{ cm}
\]
So the image is inside the tank, 24.9 cm behind the front wall; virtual, right side up, enlarged.

(b) Now we have
\[
\frac{1.33}{90\text{ cm}} + \frac{1.00}{q} = \frac{1.00 - 1.33}{-80\text{ cm}} \quad q = -93.9\text{ cm}
\]
So the image is inside the tank, 93.9 cm behind the front wall; virtual, right side up, enlarged.

(c) In case (a) the result of problem 24 gives
\[
M = -\frac{n_1 q}{\rho n_2} = -\frac{1.33(-24.9)}{1.00(30)} = -1.10
\]
In case (b) we have
\[
M = -\frac{1.33(-93.9)}{1.00(90)} = -1.39
\]
(d) In case (a) \(h' = M h = 1.10(9.00\text{ cm}) = 9.92\text{ cm}\) In case (b), the farther lobster looms larger:
\[
h' = M h = 1.30(9.00\text{ cm}) = 12.5\text{ cm}
\]
(e) The plastic has uniform thickness, so the surfaces of entry and exit for any particular ray are very nearly parallel. The ray is slightly displaced, but it would not be changed in direction by going through the plastic wall with air on both sides. Only the difference between the air and water is responsible for the refraction of the light.

P36.27 (a) \[
\frac{1}{f} = (n-1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right] = (0.440) \left[ \frac{1}{12.0\text{ cm}} - \frac{1}{-18.0\text{ cm}} \right]
\]
\[
f = \boxed{16.4\text{ cm}}
\]
We are looking at an enlarged, upright, virtual image:

\[ M = \frac{h'}{h} = 2 = -\frac{q}{p} \text{ so } p = -\frac{q}{2} = -\frac{(-2.84 \text{ cm})}{2} = +1.42 \text{ cm} \]

\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \text{ gives } \frac{1}{1.42 \text{ cm}} + \frac{1}{(-2.84 \text{ cm})} = \frac{1}{f} \]

\[ f = 2.84 \text{ cm} \]

We may differentiate through with respect to \( p \):

\[ -1p^{-2} - 1q^{-2} \frac{dq}{dp} = 0 \]

\[ \frac{dq}{dp} = -\frac{q^2}{p^2} = -M^2 \]

\[^*P36.38\] (a) \( \frac{1}{p_a} + \frac{1}{q_a} = \frac{1}{f} \) becomes \( \frac{1}{30.0 \text{ cm}} + \frac{1}{14.0 \text{ cm}} = \frac{1}{10.0 \text{ cm}} \) or \( q_a = 26.2 \text{ cm} \)

\[ h_b = hM_a = h \left( -\frac{q_b}{p_a} \right) = (10.0 \text{ cm})(-0.875) = -8.75 \text{ cm} \]

\[ \frac{1}{20.0 \text{ cm}} + \frac{1}{q_d} = \frac{1}{14.0 \text{ cm}} \text{ or } q_d = 46.7 \text{ cm} \]

\[ h_c = hM_d = (10.0 \text{ cm})(-2.33) = -23.3 \text{ cm} \]

The square is imaged as a trapezoid.
(b) \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \) becomes \( \frac{1}{p} + \frac{1}{q} = \frac{1}{14 \text{ cm}} \) or \( 1/ p = 1/ 14 \text{ cm} - 1/ q \)

\[ |h'| = |hM| = \left| h \left( \frac{-q}{p} \right) \right| = (10.0 \text{ cm}) q \left( \frac{1}{14 \text{ cm}} - \frac{1}{q} \right) \]

(c) The quantity \( \int_{q_{in}}^{q_{out}} h|dq| \) adds up the geometrical (positive) areas of thin vertical ribbons comprising the whole area of the image. We have

\[ \int_{q_{in}}^{q_{out}} h|dq| = \int_{q_{in}}^{q_{out}} (10.0 \text{ cm}) \left( \frac{q}{14 \text{ cm}} - 1 \right) dq = (10.0 \text{ cm}) \left( \frac{q^2}{28 \text{ cm}} - q \right)_{26.2 \text{ cm}}^{46.7 \text{ cm}} \]

Area = \( (10.0 \text{ cm}) \left( \frac{46.7^2 - 26.2^2}{28} - 46.7 + 26.2 \right) \text{ cm} = 328 \text{ cm}^2 \]

**P36.47** \( f_o = 20.0 \text{ m} \quad f_e = 0.025 \text{ m} \)

(a) The angular magnification produced by this telescope is \( m = -\frac{f_o}{f_e} = -800 \).

(b) Since \( m < 0 \), the image is inverted.

**P36.48** (a) The mirror-and-lens equation \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \) gives

\[ q = \frac{1}{1/f - 1/p} = \frac{1}{(p-f)/fp} = \frac{fp}{p-f} \]

Then,

\[ M = \frac{h'}{h} = \frac{-q}{p} = -\frac{f}{p-f} \]

\( h' = \frac{hf}{f-p} \)

(b) For \( p >> f \), \( f-p \approx -p \). Then,

\( h' = -\frac{hf}{p} \)

(c) Suppose the telescope observes the space station at the zenith:

\[ h' = -\frac{hf}{p} = -\frac{(108.6 \text{ m})(4.00 \text{ m})}{407 \times 10^3 \text{ m}} = -1.07 \text{ mm} \]
**P36.54** Start with the first pass through the lens.

\[
\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{\rho_1} = \frac{1}{80.0 \text{ cm}} - \frac{1}{100 \text{ cm}} \quad q_1 = 400 \text{ cm to the right of lens}
\]

For the mirror,

\[
\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{\rho_2} = \frac{1}{\rho_2} - \frac{1}{(-300 \text{ cm})} \quad q_2 = -60.0 \text{ cm}
\]

For the second pass through the lens,

\[
\frac{1}{q_3} = \frac{1}{f_1} - \frac{1}{\rho_3} = \frac{1}{80.0 \text{ cm}} - \frac{1}{160 \text{ cm}} \quad q_3 = 160 \text{ cm to the left of lens}
\]

\[
M_1 = -\frac{q_1}{\rho_1} = -\frac{400 \text{ cm}}{100 \text{ cm}} = -4.00 \quad M_2 = -\frac{q_2}{\rho_2} = -\frac{(-60.0 \text{ cm})}{(-300 \text{ cm})} = \frac{1}{5}
\]

\[
M_3 = -\frac{q_3}{\rho_3} = -\frac{160 \text{ cm}}{160 \text{ cm}} = -1 \quad M = M_1M_2M_3 = [-0.800]
\]

Since \( M < 0 \) the final image is **inverted**.

**P36.56**

\[
\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{\rho_1} = \frac{1}{10.0 \text{ cm}} - \frac{1}{12.5 \text{ cm}} \quad q_1 = 50.0 \text{ cm (to left of mirror)}
\]

This serves as an object for the lens (a virtual object), so

\[
\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{\rho_2} = \frac{1}{(-16.7 \text{ cm})} - \frac{1}{(-25.0 \text{ cm})} \quad q_2 = -50.3 \text{ cm}
\]

meaning 50.3 cm to the right of the lens. Thus, the final image is located **25.3 cm to right of mirror**.

\[
M_1 = -\frac{q_1}{\rho_1} = -\frac{50.0 \text{ cm}}{12.5 \text{ cm}} = -4.00
\]

\[
M_2 = -\frac{q_2}{\rho_2} = -\frac{(-50.3 \text{ cm})}{(-25.0 \text{ cm})} = 2.01
\]

\[
M = M_1M_2 = 8.05
\]

Thus, the final image is **virtual, upright**, 8.05 times the size of object, and 25.3 cm to right of the mirror.

**P36.59** A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light’s exit from the second surface, for which \( R = -6.00 \text{ cm} \).

The incident rays are parallel, so \( p = \infty \).
Then,
\[ \frac{n_1 + n_2}{p} + \frac{n_2 - n_1}{q} = \frac{1}{R} \]
becomes
\[ 0 + \frac{1}{q} = \frac{1.00 - 1.56}{-6.00 \text{ cm}} \]
and
\[ q = 10.7 \text{ cm} \]

P36.61  From the thin lens equation,
\[ q_1 = \frac{f_1 p_1}{p_1 - f_1} - \frac{(-6.00 \text{ cm})(12.0 \text{ cm})}{12.0 \text{ cm} - (-6.00 \text{ cm})} = -4.00 \text{ cm} \]
When we require that \( q_2 \to \infty \), the thin lens equation becomes \( p_2 = f_2 \).
In this case,
\[ p_2 = d - (-4.00 \text{ cm}) \]
Therefore,
\[ d + 4.00 \text{ cm} = f_2 = 12.0 \text{ cm} \quad \text{and} \quad d = 8.00 \text{ cm} \]

P36.64 (a) The lens makers’ equation,
\[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]
becomes:
\[ \frac{1}{5.00 \text{ cm}} = (n-1) \left( \frac{1}{9.00 \text{ cm}} - \frac{1}{(-11.0 \text{ cm})} \right) \]
giving \( n = 1.99 \).

(b) As the light passes through the lens for the first time, the thin lens equation
\[ \frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} \]
becomes:
\[ \frac{1}{8.00 \text{ cm}} + \frac{1}{q_1} = \frac{1}{5.00 \text{ cm}} \]
or \( q_1 = 13.3 \text{ cm} \), and \( M_1 = \frac{-q_1}{p_1} = \frac{-13.3 \text{ cm}}{8.00 \text{ cm}} = -1.67 \)
This image becomes the object for the concave mirror with:
\[ p_m = 20.0 \text{ cm} - q_1 = 20.0 \text{ cm} - 13.3 \text{ cm} = 6.67 \text{ cm} \]
and
\[ f = \frac{R}{2} = +4.00 \text{ cm} \]
The mirror equation becomes:
\[
\frac{1}{6.67 \text{ cm}} + \frac{1}{q_m} = \frac{1}{4.00 \text{ cm}}
\]
giving
\[q_m = 10.0 \text{ cm}\]
and
\[M_2 = -\frac{q_m}{p_m} = -\frac{10.0 \text{ cm}}{6.67 \text{ cm}} = -1.50\]

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:
\[\rho_3 = 20.0 \text{ cm} - q_m = +10.0 \text{ cm}\]
The thin lens equation yields:
\[
\frac{1}{10.0 \text{ cm}} + \frac{1}{q_3} = \frac{1}{5.00 \text{ cm}}
\]
or
\[q_3 = 10.0 \text{ cm}\]
and
\[M_3 = -\frac{q_3}{\rho_3} = -\frac{10.0 \text{ cm}}{10.0 \text{ cm}} = -1.00\]

The final image is a real image located
\[10.0 \text{ cm to the left of the lens}\]
The overall magnification is
\[M_{total} = M_1 M_2 M_3 = -2.50\]

(c) Since the total magnification is negative, this final image is inverted.

---

**P36.66**

The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).

The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror.

Thus, the image is real, inverted, and actual size.

For the upper mirror:
\[
\frac{1}{\rho} + \frac{1}{q} = \frac{1}{f} : \quad \frac{1}{7.50 \text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50 \text{ cm}} \quad q_1 = \infty
\]

For the lower mirror:
\[
\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50 \text{ cm}} \quad q_2 = 7.50 \text{ cm}
\]

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.
For lens one, as shown in the first figure,

\[
\frac{1}{q_1} = \frac{1}{120\,\text{cm}} \quad \text{and} \quad \frac{1}{q_1} = \frac{1}{40.0\,\text{cm}}
\]

\[
q_1 = 120\,\text{cm}
\]

\[
M_1 = -\frac{q_1}{\rho_1} = -\frac{120\,\text{cm}}{40.0\,\text{cm}} = -3.00
\]

This real image \( I_1 = O_2 \) is a virtual object for the second lens. That is, it is behind the lens, as shown in the second figure. The object distance is

\[
\rho_2 = 110\,\text{cm} - 120\,\text{cm} = -10.0\,\text{cm}
\]

\[
\frac{1}{q_2} = \frac{1}{-20.0\,\text{cm}}
\]

\[
q_2 = 20.0\,\text{cm}
\]

\[
M_2 = -\frac{q_2}{\rho_2} = -\frac{20.0\,\text{cm}}{-10.0\,\text{cm}} = +2.00
\]

\[
M_{\text{overall}} = M_1 M_2 = [-6.00]
\]

(b) \( M_{\text{overall}} < 0 \), so final image is \textit{inverted}.

(c) If lens two is a converging lens (third figure):

\[
\frac{1}{q_2} = \frac{1}{6.67\,\text{cm}}
\]

\[
q_2 = 6.67\,\text{cm}
\]

\[
M_2 = -\frac{6.67\,\text{cm}}{-10.0\,\text{cm}} = +0.667
\]

\[
M_{\text{overall}} = M_1 M_2 = [-2.00]
\]

Again, \( M_{\text{overall}} < 0 \) and the final image is \textit{inverted}.