Chapter 7  Energy of a System

P7.1  
(a) \[ W = F \Delta r \cos \theta = (16.0 \, \text{N})(2.20 \, \text{m}) \cos 25.0^\circ = 31.9 \, \text{J} \]

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do 0 work.

(d) \[ \sum W = 31.9 \, \text{J} + 0 + 0 = 31.9 \, \text{J} \]

P7.3  METHOD ONE

Let \( \phi \) represent the instantaneous angle the rope makes with the vertical as it is swinging up from \( \phi_i = 0 \) to \( \phi_f = 60^\circ \). In an incremental bit of motion from angle \( \phi \) to \( \phi + d\phi \), the definition of radian measure implies that \( \Delta r = (12 \, \text{m}) d\phi \). The angle \( \theta \) between the incremental displacement and the force of gravity is \( \theta = 90^\circ + \phi \). Then \( \cos \theta = \cos (90^\circ + \phi) = -\sin \phi \).

The work done by the gravitational force on Batman is

\[
W = \int F \cos \theta \, dr = \int_{\phi=0}^{\phi=60^\circ} mg(-\sin \phi)(12 \, \text{m}) \, d\phi
\]

\[
= -mg(12 \, \text{m}) \int_{0}^{60^\circ} \sin \phi \, d\phi = (-80 \, \text{kg})(9.8 \, \text{m/s}^2)(12 \, \text{m})(-\cos 60^\circ)_{0}^{60^\circ}
\]

\[
= (-784 \, \text{N})(12 \, \text{m})(-\cos 60^\circ + 1) = -4.70 \times 10^3 \, \text{J}
\]

METHOD TWO

The force of gravity on Batman is \( mg = (80 \, \text{kg})(9.8 \, \text{m/s}^2) = 784 \, \text{N} \) down. Only his vertical displacement contributes to the work gravity does. His original \( y \)-coordinate below the tree limb is \(-12 \, \text{m} \). His final \( y \)-coordinate is \(-12 \, \text{m} \cos 60^\circ = -6 \, \text{m} \). His change in elevation is \(-6 \, \text{m} - (-12 \, \text{m}) = 6 \, \text{m} \). The work done by gravity is

\[
W = F \Delta r \cos \theta = (784 \, \text{N})(6 \, \text{m}) \cos 180^\circ = -4.70 \, \text{kJ}
\]

P7.5  
\[
\mathbf{A} \cdot \mathbf{B} = (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \cdot (B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k})
\]

\[
= A_x B_x (\mathbf{i} \cdot \mathbf{i}) + A_x B_y (\mathbf{i} \cdot \mathbf{j}) + A_x B_z (\mathbf{i} \cdot \mathbf{k})
\]

\[
+ A_y B_x (\mathbf{j} \cdot \mathbf{i}) + A_y B_y (\mathbf{j} \cdot \mathbf{j}) + A_y B_z (\mathbf{j} \cdot \mathbf{k})
\]

\[
+ A_z B_x (\mathbf{k} \cdot \mathbf{i}) + A_z B_y (\mathbf{k} \cdot \mathbf{j}) + A_z B_z (\mathbf{k} \cdot \mathbf{k})
\]

\[
\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z
\]
(a) \[ W = F \cdot \Delta r = F_x \Delta x + F_y \Delta y = (6.00)(3.00) \text{N} \cdot \text{m} + (-2.00)(1.00) \text{N} \cdot \text{m} = 16.0 \text{J} \]

(b) \[ \theta = \cos^{-1} \left( \frac{F \cdot \Delta r}{F \cdot r} \right) = \cos^{-1} \left( \frac{16}{\sqrt{(6.00)^2 + (-2.00)^2}((3.00)^2 + (1.00)^2)} \right) = 36.9^\circ \]

\[ \mathbf{A} - \mathbf{B} = (3.00\hat{i} + j - \hat{k}) - (-\hat{i} + 2.00\hat{j} + 5.00\hat{k}) \]

\[ \mathbf{A} - \mathbf{B} = 4.00\hat{i} - j - 6.00\hat{k} \]

\[ \mathbf{C} \cdot (\mathbf{A} - \mathbf{B}) = (2.00\hat{j} - 3.00\hat{k}) \cdot (4.00\hat{i} - j - 6.00\hat{k}) = 0 + (-2.00) + (+18.0) = 16.0 \]

\[ W = \int \mathbf{F} \cdot d\mathbf{r} = \int_0^5 (4\hat{x} + 3\hat{y}) \text{N} \cdot d\hat{x} \]

\[ \int_0^5 (4 \text{N/m}) x dx = 0 = (4 \text{N/m}) \frac{x^2}{2} \bigg|_0^5 = 50.0 \text{J} \]

(a) Spring constant is given by \[ F = k \]

\[ k = \frac{F}{x} = \frac{(230 \text{N})}{(0.400 \text{m})} = 575 \text{N/m} \]

(b) Work \[ F_w = \frac{1}{2}(230 \text{N})(0.400 \text{m}) = 46.0 \text{J} \]

The spring exerts on each block an outward force of magnitude \[ |F_s| = kx = (3.85 \text{ N/m})(0.08 \text{ m}) = 0.308 \text{ N} \]

Take the +x direction to the right. For the light block on the left, the vertical forces are given by \[ F_y = mg = (0.25 \text{ kg})(9.8 \text{ m/s}^2) = 2.45 \text{ N} \]

\[ \sum F_y = 0 \quad n - 2.45 = 0, \quad n = 2.45 \text{ N} \]

Similarly for the heavier block \[ n = F_y = (0.5 \text{ kg})(9.8 \text{ m/s}^2) = 4.9 \text{ N} \]

(a) For the block on the left, \[ \sum F_x = ma, \quad -0.308 \quad 0.25 \text{kg} \]

\[ a = -1.23 \text{ m/s}^2 \]. For the heavier block, \[ +0.308 \quad 0.5 \text{kg} \]

\[ a = 0.616 \text{ m/s}^2 \]

(b) For the block on the left, \[ F_x = \mu_n n = 0.1(2.45 N) = 0.245 \text{ N} \]

\[ \sum F_x = ma, \quad -0.308 \text{ m/s}^2 + 0.245 \text{ N} = (0.25 \text{ kg})a \]

\[ a = -0.252 \text{ m/s}^2 \] if the force of static friction is not too large.
For the block on the right, \( f_k = \mu_k n = 0.490 \text{ N} \). The maximum force of static friction would be larger, so no motion would begin and the acceleration is zero.

(c) Left block: \( f_k = 0.462(2.45 \text{ N}) = 1.13 \text{ N} \). The maximum static friction force would be larger, so the spring force would produce no motion of this block or of the right-hand block, which could feel even more friction force. For both \( a = 0 \).

P7.25

(a) The radius to the object makes angle \( \theta \) with the horizontal, so its weight makes angle \( \theta \) with the negative side of the \( x \)-axis, when we take the \( x \)-axis in the direction of motion tangent to the cylinder.

\[
\begin{align*}
\sum F_x &= ma_x \\
F - mg \cos \theta &= 0 \\
F &= mg \cos \theta
\end{align*}
\]

(b) \( W = \int F \cdot dr \)

We use radian measure to express the next bit of displacement as \( dr = Rd\theta \) in terms of the next bit of angle moved through:

\[
W = \int_0^{\theta/2} mg \cos \theta Rd\theta = mgR \sin \theta |_0^{\theta/2}
\]

\[W = mgR(1 - 0) = \boxed{mgR}\]

P7.29

(a) \( K_A = \frac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}\)

(b) \( \frac{1}{2}mv_A^2 = K_B: v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = 5.00 \text{ m/s} \)

(c) \[
\sum W = \Delta K = K_B - K_A = \frac{1}{2}m(v_B^2 - v_A^2) = 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}
\]

P7.31 \( \vec{v}_i = (6.00\hat{i} - 2.00\hat{j}) \text{ m/s} \)

(a) \( v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0} \text{ m/s} \)

\( K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00 \text{ kg})(40.0 \text{ m}^2/\text{s}^2) = \boxed{60.0 \text{ J}} \)

(b) \( \vec{v}_f = 8.00\hat{i} + 4.00\hat{j} \)

\( v_f^2 = \vec{v}_f \cdot \vec{v}_f = 64.0 + 16.0 = 80.0 \text{ m}^2/\text{s}^2 \)

\( \Delta K = K_f - K_i = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{3.00}{2} (80.0) - 60.0 = \boxed{60.0 \text{ J}} \)
P7.36
(a) \[ v_f = 0.096 \left(3 \times 10^8 \text{ m/s}\right) = 2.88 \times 10^7 \text{ m/s} \]

\[ K_f = \frac{1}{2} mv_f^2 = \frac{1}{2} \left(9.11 \times 10^{-31} \text{ kg}\right) \left(2.88 \times 10^7 \text{ m/s}\right)^2 = 3.78 \times 10^{-16} \text{ J} \]

(b) \[ K_i + W = K_f : \quad 0 + F \Delta r \cos \theta = K_f \]
\[ F(0.028 \text{ m}) \cos 0^\circ = 3.78 \times 10^{-16} \text{ J} \]
\[ F = 1.35 \times 10^{-14} \text{ N} \]

(c) \[ \sum F = ma; \quad a = \frac{\sum F}{m} = \frac{1.35 \times 10^{-14} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = 1.48 \times 10^{16} \text{ m/s}^2 \]

(d) \[ v_f = v_i + at \quad 2.88 \times 10^7 \text{ m/s} = 0 + \left(1.48 \times 10^{16} \text{ m/s}^2\right)t \]
\[ t = 1.94 \times 10^{-9} \text{ s} \]

Check:
\[ x_f = x_i + \frac{1}{2} \left(v_i + v_f\right)t \]
\[ 0.028 \text{ m} = 0 + \frac{1}{2}(0 + 2.88 \times 10^7 \text{ m/s})t \]
\[ t = 1.94 \times 10^{-9} \text{ s} \]

P7.39
\[ F_g = mg = (4.00 \text{ kg})(9.80 \text{ m/s}^2) = 39.2 \text{ N} \]

(a) Work along OAC = work along OA + work along AC
\[ = F_g \left(\text{OA} \cos 90.0^\circ + F_g \left(\text{AC} \cos 180^\circ\right)\right) \]
\[ = (39.2 \text{ N})(5.00 \text{ m}) + (39.2 \text{ N})(5.00 \text{ m})(-1) \]
\[ = -196 \text{ J} \]

(b) Work along OBC = Work along OB + Work along BC
\[ = F_g \left(\text{OB} \cos 180^\circ + F_g \left(\text{BC} \cos 90.0^\circ\right)\right) \]
\[ = -196 \text{ J} \]

(c) Work along OC = \[ F_g \left(\text{OC} \cos 135^\circ\right) \]
\[ = (39.2 \text{ N})(5.00 \sqrt{2} \text{ m}) - \frac{1}{\sqrt{2}} \]
\[ = -196 \text{ J} \]

The results should all be the same, since gravitational forces are conservative.

P7.44
(a) \[ U = \int_0^x \left(-Ax + Bx^2\right) dx = \frac{Ax^2}{2} - \frac{Bx^3}{3} \]
\[
\Delta U = \int_{0}^{3.00 \text{ m}} Fdx = \frac{A[(3.00^2) - (2.00)^2]}{2} - \frac{B(3.00)^3 - (2.00)^3}{3} = \frac{5.00}{2}A - \frac{19.0}{3}B
\]
\[
\Delta K = \left(\frac{5.00}{2}A + \frac{19.0}{3}B\right)
\]

**P7.46**
\[
F_x = -\frac{\delta U}{\delta x} = -\frac{\delta x^2 y - 7x}{\delta x} = -(9x^2 y - 7) = 7 - 9x^2 y
\]
\[
F_y = -\frac{\delta U}{\delta y} = -\frac{\delta x^3 y - 7x}{\delta y} = -(3x^3 - 0) = -3x^3
\]

Thus, the force acting at the point \((x, y)\) is \(\vec{F} = F_x \hat{i} + F_y \hat{j} = (7 - 9x^2 y) \hat{i} - 3x^3 \hat{j}\).

**P7.49**
(a) The new length of each spring is \(\sqrt{x^2 + L^2}\), so its extension is \(\sqrt{x^2 + L^2} - L\) and the force it exerts is \(k(\sqrt{x^2 + L^2} - L)\) toward its fixed end. The \(y\) components of the two spring forces add to zero. Their \(x\) components add to
\[
\vec{F} = -2k \hat{i} \left(\sqrt{x^2 + L^2} - L\right) \frac{x}{\sqrt{x^2 + L^2}} = -2kx \hat{i} \left(1 - \frac{L}{\sqrt{x^2 + L^2}}\right).
\]

(b) Choose \(U = 0\) at \(x = 0\). Then at any point the potential energy of the system is
\[
U(x) = \int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}\right) dx = 2k \int_0^x x dx - 2kL \int_0^x \frac{x}{\sqrt{x^2 + L^2}} dx
\]
\[
U(x) = kx^2 + 2kL \left(L - \sqrt{x^2 + L^2}\right)
\]
(c) \(U(x) = 40.0x^2 + 96.0(1.20 - \sqrt{x^2 + 1.44})\)

For negative \(x\), \(U(x)\) has the same value as for positive \(x\). The only equilibrium point (i.e., where \(F_x = 0\)) is \(x = 0\).

(d) \(K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f\)
\[
0 + 0.400 \text{ J} = \frac{1}{2} (1.18 \text{ kg}) v_f^2 + 0
\]
\[
v_f = 0.823 \text{ m/s}
\]

**FIG. P7.49(c)**

**P7.51**
At start, \(\vec{v} = (40.0 \text{ m/s}) \cos 30.0^\circ \hat{i} + (40.0 \text{ m/s}) \sin 30.0^\circ \hat{j}\)
At apex, \(\vec{v} = (40.0 \text{ m/s}) \cos 30.0^\circ \hat{i} + 0 \hat{j} = (34.6 \text{ m/s}) \hat{i}\)

And \(K = \frac{1}{2}mv^2 = \frac{1}{2} (0.150 \text{ kg})(34.6 \text{ m/s})^2 = 90.0 \text{ J}\)