Find the current in the loop and use it to find the initial voltage drops across the two $RC$ circuits:

$$i = \frac{15}{75,000} = 0.2 \text{ mA}, \quad v_5(0^-) = 4 \text{ V}, \quad v_1(0^-) = 8 \text{ V}$$

There are two time constants in the circuit, one for each $RC$ subcircuit. $\tau_5$ is the time constant for the $5 \mu F - 20 \Omega$ subcircuit, and $\tau_1$ is the time constant for the $1 \mu F - 40 \Omega$ subcircuit:

$$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100 \text{ ms} \quad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40 \text{ ms}$$

Therefore,

$$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t} \text{ V}, \quad t \geq 0$$

$$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t} \text{ V}, \quad t \geq 0$$

Finally,

$$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}] \text{ V}, \quad t \geq 0$$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$$v_1(60 \text{ ms}) = 8e^{-25(0.06)} \cong 1.79 \text{ V}, \quad v_5(60 \text{ ms}) = 4e^{-10(0.06)} \cong 2.20 \text{ V}$$

$$w_1(60 \text{ ms}) = \frac{1}{2}Cv_1^2(60 \text{ ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59 \mu \text{ J}$$

$$w_5(60 \text{ ms}) = \frac{1}{2}Cv_5^2(60 \text{ ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05 \mu \text{ J}$$

$$w(60 \text{ ms}) = 1.59 + 12.05 = 13.64 \mu \text{ J}$$

Find the initial energy from the initial voltage:

$$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72 \mu \text{ J}$$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$$w_{\text{diss}} = w(0) - w(60 \text{ ms}) = 72 - 13.64 = 58.36 \mu \text{ J}$$

$$\% \text{ dissipated} = (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05 \%$$

AP 7.5 [a] Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:
\[ i(0^-) = \frac{24}{2} = 12 \text{ A} = i(0^+) \]

Note that \( i(0^-) = i(0^+) \) because the current in an inductor is continuous.

[b] Use the circuit at \( t = 0^+ \), shown below, to calculate the voltage drop across the inductor at \( 0^+ \). Note that this is the same as the voltage drop across the 10 \( \Omega \) resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.

\[ v(0^+) = -10(8 + 12) = -200 \text{ V} \]

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for \( t > 0 \). Only the 10 \( \Omega \) resistor is connected to the inductor for \( t > 0 \). Thus,

\[ \tau = \frac{L}{R} = \frac{200 \times 10^{-3}}{10} = 20 \text{ ms} \]

[d] To find \( i(t) \), we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:

\[ i_f = -8 \text{ A} \]

Now,

\[ i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02} = -8 + 20e^{-50t} \text{ A, } \quad t \geq 0 \]

[e] To find \( v(t) \), use the relationship between voltage and current for an inductor:

\[ v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t} \text{ V, } \quad t \geq 0^+ \]
P 7.33 After making a Thévenin equivalent we have

\[ I_0 = \frac{180}{15} = 12 \text{ mA} \]

\[ \tau = \left( \frac{0.25}{20} \right) \times 10^{-3} = 0.125 \times 10^{-4}; \quad \frac{1}{\tau} = 80,000 \]

\[ I_t = \frac{V_s}{R} = \frac{180}{20} = 9 \text{ mA} \]

\[ i_o = 9 + (12 - 9)e^{-80,000t} = 9 + 3e^{-80,000t} \text{ mA} \]

\[ v_o = [180 - 12(20)e^{-80,000t}] = -60e^{-80,000t} \text{ V} \]

P 7.34 \( t < 0 \) \[ i_d(0^-) \]

\[ i_L(0^-) = 6 \text{ A} \]

\( t > 0 \)

\[ i_L(\infty) = \frac{32 + 48}{20} = 4 \text{ A} \]
\[ \tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{20} = 250 \mu s; \quad \frac{1}{\tau} = 4000 \]

\[ i_L = 4 + (6 - 4)e^{-4000t} = 4 + 2e^{-4000t} \mathrm{A}, \quad t \geq 0 \]

\[ v_o = -8i_L + 48 = -8(4 + 2e^{-4000t}) + 48 = 16 - 16e^{-4000t} \mathrm{V}, \quad t \geq 0^+ \]

\[ [b] \quad v_L = 5 \times 10^{-3} \frac{di_L}{dt} = 5 \times 10^{-3}[-8000e^{-4000t}] = -40e^{-4000t} \mathrm{V}, \quad t \geq 0^+ \]

\[ v_L(0^+) = -40 \mathrm{V} \]

\[ v_o(0^+) = 0 \mathrm{V} \]

Check: at \( t = 0^+ \) the circuit is:

\[ \begin{align*}
\text{12}\Omega & \quad 6\Omega & \quad 8\Omega \\
\downarrow & \quad \circled{+} & \quad \circled{+} \\
32\mathrm{V} & \quad - & \quad v_o(0^+) \\
\circled{-} & \quad v_L(0^+) & \quad 40\mathrm{V} \\
\end{align*} \]

\[ v_L(0^+) = 32 - 72 + 0 = -40 \mathrm{V}, \quad v_o(0^+) = 48 - 48 = 0 \mathrm{V} \]

P 7.35 \[ a \] \( t < 0 \)

\[ \begin{align*}
\downarrow & \quad 5\Omega \\
\text{40A} & \quad 4\Omega & \quad \circled{+} & \quad \circled{+} & \quad 20\Omega \\
& \quad v_o(0^-) & \quad 20\Omega \\
\circled{-} & \quad - & \quad \circled{-} & \quad \circled{+} \\
\end{align*} \]

KVL equation at the top node:

\[ -40 = \frac{v_o(0^-)}{4} + \frac{v_o(0^-)}{20} + \frac{v_o(0^-)}{5} \]

Multiply by 20 and solve:

\[ -800 = (5 + 1 + 4)v_o; \quad v_o = -80 \mathrm{V} \]

\[ \therefore \quad i_o(0^-) = \frac{v_o}{5} = -80/5 = -16 \mathrm{A} \]
\[ b \] \quad i_o = \frac{v_o}{200,000} = \frac{24e^{-500t}}{200,000} = 120e^{-500t} \, \mu A

\[ v_1 = \frac{1}{15 \times 10^{-9}} \times 120 \times 10^{-6} \int_0^t e^{-500x} \, dx + 0 = 16 - 16e^{-500t} \, V, \quad t \geq 0 \]

**P 7.49** 

[a] The energy delivered to the 200 kΩ resistor is equal to the energy stored in the equivalent capacitor. From the solution to Problem 7.48 we have

\[ w = \frac{1}{2}C_o v_o^2 = \frac{1}{2}(10 \times 10^{-9})(24)^2 = 2.88 \, \mu J \]

[b] From the solution to Problem 7.48 we know the voltage on the 15 nF capacitor at \( t = \infty \) is 16 V. Therefore, the voltage across the 30 nF capacitor at \( t = \infty \) is \(-16\) V. It follows that the total energy trapped is

\[ w_{\text{trapped}} = \frac{1}{2}(30 \times 10^{-9})(-16)^2 + \frac{1}{2}(15 \times 10^{-9})(16)^2 = 5.76 \, \mu J \]

[c] \( w(0) = \frac{1}{2}(30 \times 10^{-9})(24^2) = 8.64 \, \mu J \)

Check: \( w_{\text{trapped}} + w_{\text{diss}} = 5.76 + 2.88 = 8.64 = w(0) \)

**P 7.50** 

[a] \( t > 0 \)

\begin{align*}
\quad v_o(0^-) = v_o(0^+) = 0 \, V \\
\quad v_o(\infty) = 40 \, V \\
\quad \tau = (8 \times 10^3)(25) \times 10^{-9} = 0.2 \, ms \quad 1/\tau = 5000 \\
\quad v_o = (40 - 40e^{-5000t}) \, V, \quad t \geq 0
\end{align*}

[b] \[ i_c = 25 \times 10^{-9} \frac{dv_o}{dt} \]

\[ i_c = 25 \times 10^{-9}(200,000e^{-5000t}) = 5e^{-5000t} \, mA \]

\[ v_1 = 4(5e^{-5000t}) + 40 - 40e^{-5000t} = 40 - 20e^{-5000t} \]

\[ i_o = \frac{v_1}{20 \times 10^3} = 2 - e^{-5000t} \, mA \]

[c] \[ i_1(t) = i_o + i_c = 2 + 4e^{-5000t} \, mA \]
[d] \( i_2(t) = \frac{v_1}{5 \times 10^3} = 8 - 4e^{-5000t} \text{ mA} \)

[e] \( i_1(0^+) = 2 + 4 = 6 \text{ mA} \)

Checks: \( i_1 + i_2 = 10 \text{ mA} \)

\[ i_c(0^+) = \frac{10 \left( \frac{1}{4} \right)}{\left( \frac{1}{5} + \frac{1}{20} + \frac{1}{4} \right)} = 5 \text{ mA} \]

\[ i_o(0^+) = \frac{10 \left( \frac{1}{20} \right)}{\left( \frac{1}{5} + \frac{1}{20} + \frac{1}{4} \right)} = 1 \text{ mA} \]

\[ i_1(0^+) = 5 + 1 = 6 \text{ mA} \]

P 7.51 For \( t < 0 \)

\( v_o(0^-) = v_o(0^+) = -90 \text{ V} \)

\( t > 0 \)

\[ v_o = 0.05 \mu F \]