Response of First-Order $RL$ and $RC$ Circuits

Assessment Problems

AP 7.1 [a] The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the 2 Ω resistor from the circuit.

![Circuit Diagram]

First combine the 30 Ω and 6 Ω resistors in parallel:
$30 || 6 = 5 \Omega$

Use voltage division to find the voltage drop across the parallel resistors:
$v = \frac{5}{5+3}(120) = 75 \text{ V}$

Now find the current using Ohm’s law:
$i(0^-) = \frac{v}{6} = \frac{75}{6} = -12.5 \text{ A}$

[b] $w(0) = \frac{1}{2}Lt^2(0) = \frac{1}{2}(8 \times 10^{-3})(12.5)^2 = 625 \text{ mJ}$

[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the 2 Ω resistor remains connected to the inductor. Thus,
$\tau = \frac{L}{R} = \frac{8 \times 10^{-3}}{2} = 4 \text{ ms}$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t} \text{ A, \quad t \geq 0}$

[e] $i(5 \text{ ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58 \text{ A}$
So \( w(5 \text{ ms}) = \frac{1}{2} L i^2(5 \text{ ms}) = \frac{1}{2}(8) \times 10^{-3}(3.58)^2 = 51.3 \text{ mJ} \)

\( w(\text{dis}) = 625 - 51.3 = 573.7 \text{ mJ} \)

% dissipated = \( \left( \frac{573.7}{625} \right) \times 100 = 91.8\% \)

AP 7.2 [a] First, use the circuit for \( t < 0 \) to find the initial current in the inductor:

Using current division,
\[
i(0^-) = \frac{10}{10 + 6} \times 6.4 = 4 \text{ A}
\]

Now use the circuit for \( t > 0 \) to find the equivalent resistance seen by the inductor, and use this value to find the time constant:

\[
R_{eq} = 4 \parallel (6 + 10) = 3.2 \Omega, \quad \therefore \quad \tau = \frac{L}{R_{eq}} = \frac{0.32}{3.2} = 0.1 \text{ s}
\]

Use the initial inductor current and the time constant to find the current in the inductor:
\[
i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t} \text{ A}, \quad t \geq 0
\]

Use current division to find the current in the 10Ω resistor:
\[
i_o(t) = \frac{4}{4 + 10 + 6} \times i = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t} \text{ A}, \quad t \geq 0^+
\]

Finally, use Ohm's law to find the voltage drop across the 10Ω resistor:
\[
v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t} \text{ V}, \quad t \geq 0^+
\]

[b] The initial energy stored in the inductor is
\[
w(0) = \frac{1}{2} Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56 \text{ J}
\]

Find the energy dissipated in the 4Ω resistor by integrating the power over all time:
\[
v_{4\Omega}(t) = L \frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t} \text{ V}, \quad t \geq 0^+
\]
\[ p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t} \text{ W}, \quad t \geq 0^+ \]

\[ w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t} \, dt = 2.048 \text{ J} \]

Find the percentage of the initial energy in the inductor dissipated in the 4 \(\Omega\) resistor:

\[ \% \text{ dissipated} = \left(\frac{2.048}{2.56}\right) 100 = 80\% \]

AP 7.3 [a] The circuit for \(t < 0\) is shown below. Note that the capacitor behaves like an open circuit.

Find the voltage drop across the open circuit by finding the voltage drop across the 50 k\(\Omega\) resistor. First use current division to find the current through the 50 k\(\Omega\) resistor:

\[ i_{50k} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3} (7.5 \times 10^{-3}) = 4 \text{ mA} \]

Use Ohm's law to find the voltage drop:

\[ v(0^-) = (50 \times 10^3) i_{50k} = (50 \times 10^3)(0.004) = 200 \text{ V} \]

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for \(t > 0\). When the switch opens, only the 50 k\(\Omega\) resistor remains connected to the capacitor. Thus,

\[ \tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20 \text{ ms} \]

[c] \[ v(t) = v(0^-) e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t} \text{ V}, \quad t \geq 0 \]

[d] \[ w(0) = \frac{1}{2} C v^2 = \frac{1}{2} (0.4 \times 10^{-6})(200)^2 = 8 \text{ mJ} \]

[e] \[ w(t) = \frac{1}{2} C v^2(t) = \frac{1}{2} (0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t} \text{ mJ} \]

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

\[ 8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \quad e^{100t} = 4, \quad t = (\ln 4)/100 = 13.86 \text{ ms} \]

AP 7.4 [a] This circuit is actually two \(RC\) circuits in series, and the requested voltage, \(v_o\), is the sum of the voltage drops for the two \(RC\) circuits. The circuit for \(t < 0\) is shown below:
Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:
\[
i = \frac{15}{75,000} = 0.2 \text{ mA,} \quad v_5(0^-) = 4 \text{ V,} \quad v_1(0^-) = 8 \text{ V}
\]

There are two time constants in the circuit, one for each RC subcircuit. \(\tau_5\) is the time constant for the \(5 \mu F - 20 \Omega\) subcircuit, and \(\tau_1\) is the time constant for the \(1 \mu F - 40 \Omega\) subcircuit:
\[
\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100 \text{ ms;} \quad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40 \text{ ms}
\]

Therefore,
\[
v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t} \text{ V,} \quad t \geq 0
\]
\[
v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t} \text{ V,} \quad t \geq 0
\]

Finally,
\[
v_0(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}] \text{ V,} \quad t \geq 0
\]

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:
\[
v_1(60 \text{ ms}) = 8e^{-25(0.06)} \cong 1.79 \text{ V,} \quad v_5(60 \text{ ms}) = 4e^{-10(0.06)} \cong 2.20 \text{ V}
\]
\[
w_1(60 \text{ ms}) = \frac{1}{2}Cv_1^2(60 \text{ ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59 \mu J
\]
\[
w_5(60 \text{ ms}) = \frac{1}{2}Cv_5^2(60 \text{ ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05 \mu J
\]
\[
w(60 \text{ ms}) = 1.59 + 12.05 = 13.64 \mu J
\]

Find the initial energy from the initial voltage:
\[
w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72 \mu J
\]

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:
\[
w_{\text{diss}} = w(0) - w(60 \text{ ms}) = 72 - 13.64 = 58.36 \mu J
\]
\[
\% \text{ dissipated} = \frac{58.36 \times 10^{-6}/72 \times 10^{-6}(100) = 81.05 \%}{}
\]

AP 7.5 [a] Use the circuit at \(t < 0\), shown below, to calculate the initial current in the inductor: