AP 4.15 [a] Redraw the circuit with a helpful voltage and current labeled:

![Circuit Diagram]

Transform the 120 V source in series with the 20 Ω resistor into a 6 A source in parallel with the 20 Ω resistor. Also transform the −60 V source in series with the 5 Ω resistor into a −12 A source in parallel with the 5 Ω resistor. The result is the following circuit:

![Transformed Circuit Diagram]

Combine the three current sources into a single current source, using KCL, and combine the 20 Ω, 5 Ω, and 6 Ω resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 Ω resistor into a 72 V source in series with the 2.4 Ω resistor. Combine the 2.4 Ω resistor in series with the 1.6 Ω resistor to get a very simple circuit that still maintains the voltage \( v \). The resulting circuit is on the right.

![Simplified Circuit Diagram]

Use voltage division in the circuit on the right to calculate \( v \) as follows:

\[ v = \frac{8}{12} (72) = 48 \text{ V} \]

[b] Calculate \( i \) in the circuit on the right using Ohm’s law:

\[ i = \frac{v}{8} = \frac{48}{8} = 6 \text{ A} \]
CHAPTER 4. Techniques of Circuit Analysis

Now use \( i \) to calculate \( v_a \) in the circuit on the left:

\[
v_a = 6(1.6 + 8) = 57.6 \text{ V}
\]

Returning back to the original circuit, note that the voltage \( v_a \) is also the voltage drop across the series combination of the 120 V source and 20 \( \Omega \) resistor. Use this fact to calculate the current in the 120 V source, \( i_a \):

\[
i_a = \frac{120 - v_a}{20} = \frac{120 - 57.6}{20} = 3.12 \text{ A}
\]

\[
\rho_{120V} = -(120)i_a = -(120)(3.12) = -374.4 \text{ W}
\]

Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find \( R_{Th} \), replace the 72 V source with a short circuit:

![Circuit Diagram]

Note that the 5 \( \Omega \) and 20 \( \Omega \) resistors are in parallel, with an equivalent resistance of \( 5 \parallel 20 = 4 \Omega \). The equivalent 4 \( \Omega \) resistance is in series with the 8 \( \Omega \) resistor for an equivalent resistance of \( 4 + 8 = 12 \Omega \). Finally, the 12 \( \Omega \) equivalent resistance is in parallel with the 12 \( \Omega \) resistor, so

\( R_{Th} = 12 \parallel 12 = 6 \Omega \).

Use node voltage analysis to find \( v_{Th} \). Begin by redrawing the circuit and labeling the node voltages:

![Circuit Diagram]

The node voltage equations are

\[
\begin{align*}
\frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{Th}}{8} &= 0 \\
\frac{v_{Th} - v_1}{8} + \frac{v_{Th} - 72}{12} &= 0
\end{align*}
\]
Place these equations in standard form:

\[
\begin{align*}
v_1 \left( \frac{1}{5} + \frac{1}{20} + \frac{1}{8} \right) + v_{Th} \left( \frac{1}{8} \right) &= \frac{72}{5} \\
v_1 \left( \frac{1}{8} \right) + v_{Th} \left( \frac{1}{8} + \frac{1}{12} \right) &= 6
\end{align*}
\]

Solving, \( v_1 = 60 \) V and \( v_{Th} = 64.8 \) V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a 6 \( \Omega \) resistor.

**AP 4.17** We begin by performing a source transformation, turning the parallel combination of the 15 A source and 8 \( \Omega \) resistor into a series combination of a 120 V source and an 8 \( \Omega \) resistor, as shown in the figure on the left. Next, combine the 2 \( \Omega \), 8 \( \Omega \) and 10 \( \Omega \) resistors in series to give an equivalent 20 \( \Omega \) resistance. Then transform the series combination of the 120 V source and the 20 \( \Omega \) equivalent resistance into a parallel combination of a 6 A source and a 20 \( \Omega \) resistor, as shown in the figure on the right.

Finally, combine the 20 \( \Omega \) and 12 \( \Omega \) parallel resistors to give \( R_N = 20 \parallel 12 = 7.5 \Omega \). Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a 7.5 \( \Omega \) resistor.

**AP 4.18** Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the \(-36\) V source and 12 k\( \Omega \) resistor into a parallel combination of a \(-3\) mA source and 12 k\( \Omega \) resistor. The resulting circuit is shown below:

Now combine the two parallel current sources and the two parallel resistors to give a \(-3 + 18 = 15\) mA source in parallel with a 12 k\( \Omega \parallel 60\) k\( \Omega = 10\) k\( \Omega \) resistor. Then transform the 15 mA source in parallel with the 10 k\( \Omega \) resistor into a 150 V source in series with a 10 k\( \Omega \) resistor, and combine this 10 k\( \Omega \) resistor in series with the 15 k\( \Omega \) resistor. The Thévenin equivalent is thus a 150 V
and apply the test voltage $v_T$, as shown in the circuit below:

Write a KCL equation at the middle node:

$$i_T = i_x + 3i_x + v_T/2 = 4i_x + v_T/2$$

Use Ohm's law to determine $i_x$ as a function of $v_T$:

$$i_x = v_T/8$$

Substitute the second equation into the first equation:

$$i_T = 4(v_T/8) + v_T/2 = v_T$$

Thus,

$$R_{Th} = v_T/i_T = 1 \Omega$$

The Thévenin equivalent is an 8 V source in series with a 1 Ω resistor.

AP 4.20 Begin by calculating the open circuit voltage, which is also $v_{Th}$, using the node voltage method in the circuit below:

The node voltage equations are

$$\frac{v}{60} + \frac{v - (v_{Th} + 160i_\Delta)}{20} - 4 = 0, $$

$$\frac{v_{Th}}{40} + \frac{v_{Th}}{80} + \frac{v_{Th} + 160i_\Delta - v}{20} = 0$$

The dependent source constraint equation is

$$i_\Delta = \frac{v_{Th}}{40}$$
Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

\[ v \left( \frac{1}{60} + \frac{1}{20} \right) + v_{Th} \left( -\frac{5}{20} \right) = 4 \]
\[ v \left( -\frac{1}{20} \right) + v_{Th} \left( \frac{1}{40} + \frac{1}{80} + \frac{5}{20} \right) = 0 \]

Solving, \( v = 172.5 \text{ V} \) and \( v_{Th} = 30 \text{ V} \).

Now use the test source method to calculate the test current and thus \( R_{Th} \). Replace the current source with a short circuit and apply the test source to get the following circuit:

Write a KCL equation at the rightmost node:

\[ i_T = \frac{v_T}{80} + \frac{v_T}{40} + \frac{v_T + 160i_\Delta}{80} \]

The dependent source constraint equation is

\[ i_\Delta = \frac{v_T}{40} \]

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

\[ i_T = \frac{v_T}{10} \]

Therefore,

\[ R_{Th} = \frac{v_T}{i_T} = 10 \Omega \]

Thus, the Thévenin equivalent is a 30 V source in series with a 10 Ω resistor.

AP 4.21 First find the Thévenin equivalent circuit. To find \( v_{Th} \), create an open circuit between nodes a and b and use the node voltage method with the circuit.