Circuit Variables

Assessment Problems

AP 1.1 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing $10 \text{ billion}$ in scientific notation:

$$100 \text{ billion} = 100 \times 10^9$$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ mins}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} \cdot \frac{1 \text{ sec}}{1000 \text{ ms}} = \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{100 \times 10^9}{1 \text{ year}} \cdot \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}} = \frac{100}{31.5576} = 3.17/\text{ms}$$

AP 1.2 First, we recognize that $1 \text{ ns} = 10^{-9} \text{ s}$. The question then asks how far a signal will travel in $10^{-9} \text{ s}$ if it is traveling at $80\%$ of the speed of light. Remember that the speed of light $c = 3 \times 10^8 \text{ m/s}$. Therefore, $80\%$ of $c$ is $(0.8)(3 \times 10^8) = 2.4 \times 10^8 \text{ m/s}$. Now, we use a product of ratios to convert from meters/second to inches/nanosecond:

$$\frac{2.4 \times 10^8 \text{ m}}{1 \text{ s}} \cdot \frac{1 \text{ s}}{10^9 \text{ ns}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{(2.4 \times 10^8)(100)}{(10^9)(2.54)} = 9.45 \text{ in}$$

Thus, a signal traveling at $80\%$ of the speed of light will travel $9.45''$ in a nanosecond.
AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or 
\[ i = \frac{dq}{dt} \]. In this problem, we are given the current and asked to find the total 
charge. To do this, we must integrate Eq. (1.2) to find an expression for 
charge in terms of current:

\[ q(t) = \int_0^t i(x) \, dx \]

We are given the expression for current, \( i \), which can be substituted into the 
above expression. To find the total charge, we let \( t \to \infty \) in the integral. Thus 
we have

\[
q_{\text{total}} = \int_0^\infty 20e^{-5000x} \, dx = \left. \frac{20}{-5000} e^{-5000x} \right|_0^\infty = \frac{20}{-5000} (e^{-\infty} - e^0) \\
= \frac{20}{-5000} (0 - 1) = \frac{20}{5000} = 0.004 \text{ C} = 4000 \mu\text{C}
\]

AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or 
\[ i = \frac{dq}{dt} \]. In this problem we are given an expression for the charge, and asked to 
find the maximum current. First we will find an expression for the current 
using Eq. (1.2):

\[
i = \frac{dq}{dt} = \frac{d}{dt} \left[ \frac{1}{\alpha^2} - \left( \frac{t}{\alpha} + \frac{1}{\alpha^2} \right) e^{-\alpha t} \right] \\
= \frac{d}{dt} \left( \frac{1}{\alpha^2} \right) - \frac{d}{dt} \left( \frac{t}{\alpha} e^{-\alpha t} \right) - \frac{d}{dt} \left( \frac{1}{\alpha^2} e^{-\alpha t} \right) \\
= 0 - \left( \frac{1}{\alpha} e^{-\alpha t} - \frac{t}{\alpha} e^{-\alpha t} \right) - \left( -\alpha \frac{1}{\alpha^2} e^{-\alpha t} \right) \\
= \left( -\frac{1}{\alpha} + \frac{1}{\alpha} + \frac{1}{\alpha} \right) e^{-\alpha t} \\
= te^{-\alpha t}
\]

Now that we have an expression for the current, we can find the maximum 
value of the current by setting the first derivative of the current to zero and 
solving for \( t \):

\[
\frac{di}{dt} = \frac{d}{dt} (te^{-\alpha t}) = e^{-\alpha t} + t(-\alpha)e^{\alpha t} = (1 - \alpha t)e^{-\alpha t} = 0
\]

Since \( e^{-\alpha t} \) never equals 0 for a finite value of \( t \), the expression equals 0 only 
when \( (1 - \alpha t) = 0 \). Thus, \( t = 1/\alpha \) will cause the current to be maximum. For 
this value of \( t \), the current is

\[
i = \frac{1}{\alpha} e^{-\alpha/\alpha} = \frac{1}{\alpha} e^{-1}
\]
\[ \frac{4 \times 10^9 \text{ bytes}}{(32)(24)(2.1) \text{ mm}^3} = \frac{x \times 10^6 \text{ MB}}{(0.1)^3 \text{ mm}^3} \]
\[ x = \frac{(4 \times 10^9)(0.001)}{(32)(24)(2.1)} = 2480 \text{ bytes} \]

P 1.4
\[ \frac{(320)(240) \text{ pixels}}{1 \text{ frame}} \cdot \frac{1 \text{ pixel}}{1 \text{ sec}} \cdot \frac{30 \text{ frames}}{1 \text{ sec}} = 4.608 \times 10^6 \text{ bytes/sec} \]

\[(4.608 \times 10^6 \text{ bytes/sec})(x \text{ sec}) = 30 \times 10^9 \text{ bytes} \]
\[ x = \frac{30 \times 10^9}{4.608 \times 10^6} = 6510 \text{ sec} = 108.5 \text{ min of video} \]

P 1.5 [a] We can set up a ratio to determine how long it takes the bamboo to grow 10 $\mu$m. First, recall that 1 mm = 10$^4$ $\mu$m. Let’s also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

\[ \frac{250 \text{ mm}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456} \text{ mm/s} \]

Use a ratio to determine the time it takes for the bamboo to grow 10 $\mu$m:

\[ \frac{10}{3456} \times 10^{-3} \text{ m}}{1 \text{ s}} = \frac{10 \times 10^{-6} \text{ m}}{x \text{ s}} \quad \text{so} \quad x = \frac{10 \times 10^{-6}}{10/3456 \times 10^{-3}} = 3.456 \text{ s} \]

[b] \[ \frac{1 \text{ cell}}{3.456 \text{ s}} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} \cdot \frac{(24)(7) \text{ hr}}{1 \text{ week}} = 175,000 \text{ cells/week} \]

P 1.6 Volume = area $\times$ thickness

Convert values to millimeters, noting that 10 m$^2$ = 10$^6$ mm$^2$

\[ 10^6 = (10 \times 10^6) \text{(thickness)} \]

$\Rightarrow$ thickness $= \frac{10^6}{10 \times 10^6} = 0.10 \text{ mm} \]

P 1.7 \[ \frac{\text{C/m}^3}{1 \text{ electron}} = \frac{1.6022 \times 10^{-19} \text{ C}}{1 \text{ m}^3} \times \frac{10^{29} \text{ electrons}}{1 \text{ m}^3} = 1.6022 \times 10^{10} \text{ C/m}^3 \]

Cross-sectional area of wire = $\pi r^2 = \pi (1.5 \times 10^{-3} \text{ m})^2 = 7.07 \times 10^{-6} \text{ m}^2$

\[ \text{C/m} = (1.6022 \times 10^{10} \text{ C/m}^3)(7.07 \times 10^{-6} \text{ m}^2) = 113.253 \times 10^3 \text{ C/m} \]

Therefore, $i \left( \frac{\text{C}}{\text{sec}} \right) = (113.253 \times 10^3) \left( \frac{\text{C}}{\text{m}} \right) \times \text{avg vel} \left( \frac{\text{m}}{\text{s}} \right) \]

Thus, average velocity $= \frac{i}{113.253 \times 10^3} = \frac{1200}{113.253 \times 10^3} = 0.0106 \text{ m/s} \]


\[ \frac{4 \times 10^9 \text{ bytes}}{(32)(24)(2.1) \text{ mm}^3} = \frac{x \times 10^6 \text{ MB}}{(0.1)^3 \text{ mm}^3} \]

\[ x = \frac{(4 \times 10^9)(0.001)}{(32)(24)(2.1)} = 2480 \text{ bytes} \]

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\[ \frac{(320)(240) \text{ pixels}}{1 \text{ frame}} \cdot \frac{2 \text{ bytes}}{1 \text{ pixel}} \cdot \frac{30 \text{ frames}}{1 \text{ sec}} = 4.608 \times 10^6 \text{ bytes/sec} \]

\[ (4.608 \times 10^6 \text{ bytes/sec})(x \text{ secs}) = 30 \times 10^9 \text{ bytes} \]

\[ x = \frac{30 \times 10^9}{4.608 \times 10^6} = 6510 \text{ sec} = 108.5 \text{ min of video} \]

P 1.5

[a] We can set up a ratio to determine how long it takes the bamboo to grow 10 \( \mu \text{m} \). First, recall that 1 mm = 10^3 \( \mu \text{m} \). Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

\[
\frac{250 \text{ mm}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456} \text{ mm/s}
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Convert values to millimeters, noting that 10 \text{ m}^2 = 10^6 \text{ mm}^2

\[ 10^6 = (10 \times 10^6)(\text{thickness}) \]

\[ \Rightarrow \text{thickness} = \frac{10^6}{10 \times 10^6} = 0.10 \text{ mm} \]

P 1.7

\[ C/\text{m}^3 = \frac{1.6022 \times 10^{-19} \text{ C}}{1 \text{ electron}} \times \frac{10^{29} \text{ electrons}}{1 \text{ m}^3} = 1.6022 \times 10^{10} \text{ C/m}^3 \]

Cross-sectional area of wire = \( \pi r^2 = \pi(1.5 \times 10^{-3} \text{ m})^2 = 7.07 \times 10^{-6} \text{ m}^2 \)

\[ C/\text{m} = (1.6022 \times 10^{10} \text{ C/m}^3)(7.07 \times 10^{-6} \text{ m}^2) = 113.253 \times 10^3 \text{ C/m} \]

Therefore, \[ i \left( \frac{C}{\text{sec}} \right) = (113.253 \times 10^3) \left( \frac{C}{\text{m}} \right) \times \text{avg vel} \left( \frac{\text{m}}{\text{s}} \right) \]

Thus, average velocity = \[ \frac{i}{113.253 \times 10^3} = \frac{1200}{113.253 \times 10^3} = 0.0106 \text{ m/s} \]
P 1.8  \[ n = \frac{35 \times 10^{-6} \text{ C/s}}{1.6022 \times 10^{-19} \text{ C/elec}} = 2.18 \times 10^{14} \text{ elec/s} \]

P 1.9  First we use Eq. (1.2) to relate current and charge:

\[ i = \frac{dq}{dt} = 24 \cos 4000t \]

Therefore, \( dq = 24 \cos 4000t \, dt \)

To find the charge, we can integrate both sides of the last equation. Note that we substitute \( x \) for \( q \) on the left side of the integral, and \( y \) for \( t \) on the right side of the integral:

\[ \int_{q(0)}^{q(t)} dx = 24 \int_0^t \cos 4000y \, dy \]

We solve the integral and make the substitutions for the limits of the integral, remembering that \( \sin 0 = 0 \):

\[ q(t) - q(0) = 24 \frac{\sin 4000y}{4000} \bigg|_0^t = \frac{24}{4000} \sin 4000t - \frac{24}{4000} \sin 4000(0) = \frac{24}{4000} \sin 4000t \]

But \( q(0) = 0 \) by hypothesis, i.e., the current passes through its maximum value at \( t = 0 \), so \( q(t) = 6 \times 10^{-3} \sin 4000t \text{ C} = 6 \sin 4000t \text{ mC} \)

P 1.10  \[ w = qV = (1.6022 \times 10^{-19})(6) = 9.61 \times 10^{-19} = 0.961 \text{ aJ} \]

P 1.11  \[ p = (9)(100 \times 10^{-3}) = 0.9 \text{ W}; \quad 5 \text{ hr} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} = 18,000 \text{ s} \]

\[ w(t) = \int_0^t p \, dt \quad w(18,000) = \int_0^{18,000} 0.9 \, dt = 0.9(18,000) = 16.2 \text{ kJ} \]

P 1.12  Assume we are standing at box A looking toward box B. Then, using the passive sign convention \( p = vi \), since the current \( i \) is flowing into the + terminal of the voltage \( v \). Now we just substitute the values for \( v \) and \( i \) into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a]  \( p = (120)(5) = 600 \text{ W} \quad 600 \text{ W from A to B} \)

[b]  \( p = (250)(-8) = -2000 \text{ W} \quad 2000 \text{ W from B to A} \)

[c]  \( p = (-150)(16) = -2400 \text{ W} \quad 2400 \text{ W from B to A} \)

[d]  \( p = (-480)(-10) = 4800 \text{ W} \quad 4800 \text{ W from A to B} \)
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[c] \( p = (-150)(16) = -2400 \text{ W} \quad 2400 \text{ W from B to A} \)

[d] \( p = (-480)(-10) = 4800 \text{ W} \quad 4800 \text{ W from A to B} \)
P 1.13  [a]  
\[ p = vi = (40)(-10) = -400 \text{ W} \]
Power is being delivered by the box.
[b] Leaving
[c] Gaining

P 1.14  [a]  
\[ p = vi = (-60)(-10) = 600 \text{ W}, \text{ so power is being absorbed by the box.} \]
[b] Entering
[c] Losing

P 1.15  [a] In Car A, the current \( i \) is in the direction of the voltage drop across the 12 V battery (the current \( i \) flows into the + terminal of the battery of Car A). Therefore using the passive sign convention, 
\[ p = vi = (30)(12) = 360 \text{ W}. \]
Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.
[b] \( w(t) = \int_0^t p \, dx; \quad 1 \text{ min} = 60 \text{ s} \)
\[ w(60) = \int_0^{60} 360 \, dx \]
\[ w = 360(60 - 0) = 360(60) = 21,600 \text{ J} = 21.6 \text{ kJ} \]

P 1.16  \( p = vi; \quad w = \int_0^t p \, dx \)
Since the energy is the area under the power vs. time plot, let us plot \( p \) vs. \( t \).

Note that in constructing the plot above, we used the fact that 80 hr
\[ = 288,000 \text{ s} = 288 \text{ ks} \]
\[ p(0) = (9)(20 \times 10^{-3}) = 180 \times 10^{-3} \text{ W} \]