**Learning Objectives**

After studying this chapter you should be able to do the following:

1. Define engineering stress and engineering strain.
2. State Hooke's law, and note the conditions under which it is valid.
3. Define Poisson's ratio.
4. Given an engineering stress–strain diagram, determine (a) the modulus of elasticity, (b) the yield strength (0.002 strain offset), and (c) the tensile strength, and (d) estimate the percent elongation.
5. For the tensile deformation of a ductile cylindrical specimen, describe changes in specimen profile to the point of fracture.
6. Compute ductility in terms of both percent elongation and percent reduction of area for a material that is loaded in tension to fracture.
7. Name the two most common hardness-testing techniques; note two differences between them.
8. (a) Name and briefly describe the two different microhardness testing techniques, and (b) cite situations for which these techniques are generally used.
9. Compute the working stress for a ductile material.

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### 6.1 Introduction

Many materials, when in service, are subjected to forces or loads; examples include the aluminum alloy from which an airplane wing is constructed and the steel in an automobile axle. In such situations it is necessary to know the characteristics of the material and to design the member from which it is made such that any resulting deformation will not be excessive and fracture will not occur. The mechanical behavior of a material reflects the relationship between its response or deformation to an applied load or force. Important mechanical properties are strength, hardness, ductility, and stiffness.

The mechanical properties of materials are ascertained by performing carefully designed laboratory experiments that replicate as nearly as possible the service conditions. Factors to be considered include the nature of the applied load and its duration, as well as the environmental conditions. It is possible for the load to be tensile, compressive, or shear, and its magnitude may be constant with time, or it may fluctuate continuously. Application time may be only a fraction of a second, or it may extend over a period of many years. Service temperature may be an important factor.

Mechanical properties are of concern to a variety of parties (e.g., producers and consumers of materials, research organizations, government agencies) that have differing interests. Consequently, it is imperative that there be some consistency in the manner in which tests are conducted, and in the interpretation of their results. This consistency is accomplished by using standardized testing techniques. Establishment and publication of these standards are often coordinated by professional societies. In the United States the most active organization is the American Society for Testing and Materials (ASTM). Its *Annual Book of ASTM Standards* comprises numerous volumes, which are issued and updated yearly; a large number of these standards relate to mechanical testing techniques. Several of these are referenced by footnote in this and subsequent chapters.

The role of structural engineers is to determine stresses and stress distributions within members that are subjected to well-defined loads. This may be accomplished by experimental testing techniques and/or by theoretical and mathematical stress analyses. These topics are treated in traditional stress analysis and strength of materials texts.

Materials and metallurgical engineers, on the other hand, are concerned with producing and fabricating materials to meet service requirements as predicted by these stress analyses. This necessarily involves an understanding of the relationships...
between the microstructure (i.e., internal features) of materials and their mechanical properties.

Materials are frequently chosen for structural applications because they have desirable combinations of mechanical characteristics. The present discussion is confined primarily to the mechanical behavior of metals; polymers and ceramics are treated separately because they are, to a large degree, mechanically dissimilar to metals. This chapter discusses the stress–strain behavior of metals and the related mechanical properties, and also examines other important mechanical characteristics. Discussions of the microscopic aspects of deformation mechanisms and methods to strengthen and regulate the mechanical behavior of metals are deferred to later chapters.

### 6.2 Concepts of Stress and Strain

If a load is static or changes relatively slowly with time and is applied uniformly over a cross section or surface of a member, the mechanical behavior may be ascertained by a simple stress–strain test; these are most commonly conducted for metals at room temperature. There are three principal ways in which a load may be applied: namely, tension, compression, and shear (Figures 6.1a, b, c). In engineering

*Figure 6.1*

(a) Schematic illustration of how a tensile load produces an elongation and positive linear strain. Dashed lines represent the shape before deformation; solid lines, after deformation. (b) Schematic illustration of how a compressive load produces contraction and a negative linear strain. (c) Schematic representation of shear strain \( \gamma \), where \( \gamma = \tan \theta \). (d) Schematic representation of torsional deformation (i.e., angle of twist \( \phi \)) produced by an applied torque \( T \).
practice many loads are torsional rather than pure shear; this type of loading is illustrated in Figure 6.1d.

**TENSION TESTS**

One of the most common mechanical stress–strain tests is performed in tension. As will be seen, the tension test can be used to ascertain several mechanical properties of materials that are important in design. A specimen is deformed, usually to fracture, with a gradually increasing tensile load that is applied uniaxially along the long axis of a specimen. A standard tensile specimen is shown in Figure 6.2. Normally, the cross section is circular, but rectangular specimens are also used. During testing, deformation is confined to the narrow center region, which has a uniform cross section along its length. The standard diameter is approximately 12.8 mm (0.5 in.), whereas the reduced section length should be at least four times this diameter; 60 mm (2.4 in.) is common. Gauge length is used in ductility computations, as discussed in Section 6.6; the standard value is 50 mm (2.0 in.). The specimen is mounted by its ends into the holding grips of the testing apparatus (Figure 6.3). The tensile testing machine is designed to elongate the specimen at a constant rate, and to continuously and simultaneously measure the instantaneous applied load (with a load cell) and the resulting elongations (using an extensometer). A stress–strain test typically takes several minutes to perform and is destructive; that is, the test specimen is permanently deformed and usually fractured.

The output of such a tensile test is recorded on a strip chart (or by a computer) as load or force versus elongation. These load–deformation characteristics are dependent on the specimen size. For example, it will require twice the load to produce the same elongation if the cross-sectional area of the specimen is doubled. To minimize these geometrical factors, load and elongation are normalized to the respective parameters of engineering stress and engineering strain. Engineering stress $\sigma$ is defined by the relationship

$$\sigma = \frac{F}{A_0}$$

where $F$ is the instantaneous load applied perpendicular to the specimen cross section, in units of newtons (N) or pounds force (lb), and $A_0$ is the original cross-sectional area before any load is applied (m$^2$ or in.$^2$). The units of engineering stress (referred to subsequently as just stress) are megapascals, MPa (SI) (where 1 MPa = 10$^6$ N/m$^2$), and pounds force per square inch, psi (Customary U.S.).

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2. Conversion from one system of stress units to the other is accomplished by the relationship 145 psi = 1 MPa.
Engineering strain $\epsilon$ is defined according to

$$\epsilon = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

in which $l_0$ is the original length before any load is applied, and $l$ is the instantaneous length. Sometimes the quantity $l - l_0$ is denoted as $\Delta l$, and is the deformation elongation or change in length at some instant, as referenced to the original length. Engineering strain (subsequently called just strain) is unitless, but meters per meter or inches per inch are often used; the value of strain is obviously independent of the unit system. Sometimes strain is also expressed as a percentage, in which the strain value is multiplied by 100.

**COMPRESSION TESTS$^3$**

Compression stress–strain tests may be conducted if in-service forces are of this type. A compression test is conducted in a manner similar to the tensile test, except that the force is compressive and the specimen contracts along the direction of the stress. Equations 6.1 and 6.2 are utilized to compute compressive stress and strain, respectively. By convention, a compressive force is taken to be negative, which yields a negative stress. Furthermore, since $l_0$ is greater than $l$, compressive strains computed from Equation 6.2 are necessarily also negative. Tensile tests are more common because they are easier to perform; also, for most materials used in structural applications, very little additional information is obtained from compressive tests. Compressive tests are used when a material’s behavior under large and permanent (i.e., plastic) strains is desired, as in manufacturing applications, or when the material is brittle in tension.

SHEAR AND TORSIONAL TESTS

For tests performed using a pure shear force as shown in Figure 6.1c, the shear stress $\tau$ is computed according to

$$\tau = \frac{F}{A_0}$$

(6.3)

where $F$ is the load or force imposed parallel to the upper and lower faces, each of which has an area of $A_0$. The shear strain $\gamma$ is defined as the tangent of the strain angle $\theta$, as indicated in the figure. The units for shear stress and strain are the same as for their tensile counterparts.

Torsion is a variation of pure shear, wherein a structural member is twisted in the manner of Figure 6.1d; torsional forces produce a rotational motion about the longitudinal axis of one end of the member relative to the other end. Examples of torsion are found for machine axles and drive shafts, and also for twist drills. Torsional tests are normally performed on cylindrical solid shafts or tubes. A shear stress $\tau$ is a function of the applied torque $T$, whereas shear strain $\gamma$ is related to the angle of twist, $\phi$ in Figure 6.1d.

GEOMETRIC CONSIDERATIONS OF THE STRESS STATE

Stresses that are computed from the tensile, compressive, shear, and torsional force states represented in Figure 6.1 act either parallel or perpendicular to planar faces of the bodies represented in these illustrations. It should be noted that the stress state is a function of the orientations of the planes upon which the stresses are taken to act. For example, consider the cylindrical tensile specimen of Figure 6.4 that is subjected to a tensile stress $\sigma$ applied parallel to its axis. Furthermore, consider also the plane $p-p'$ that is oriented at some arbitrary angle $\theta$ relative to the plane of the specimen end-face. Upon this plane $p-p'$, the applied stress is no longer a pure tensile one. Rather, a more complex stress state is present that consists

---

**Figure 6.4** Schematic representation showing normal ($\sigma'$) and shear ($\tau'$) stresses that act on a plane oriented at an angle $\theta$ relative to the plane taken perpendicular to the direction along which a pure tensile stress ($\sigma$) is applied.

---

1 ASTM Standard E 143, "Standard Test for Shear Modulus."
of a tensile (or normal) stress \( \sigma' \) that acts normal to the \( p-p' \) plane, and, in addition, a shear stress \( \tau' \) that acts parallel to this plane; both of these stresses are represented in the figure. Using mechanics of materials principles, it is possible to develop equations for \( \sigma' \) and \( \tau' \) in terms of \( \sigma \) and \( \theta \), as follows:

\[
\sigma' = \sigma \cos^2 \theta = \sigma \left( \frac{1 + \cos 2\theta}{2} \right) \quad (6.4a)
\]

\[
\tau' = \sigma \sin \theta \cos \theta = \sigma \left( \frac{\sin 2\theta}{2} \right) \quad (6.4b)
\]

These same mechanics principles allow the transformation of stress components from one coordinate system to another coordinate system that has a different orientation. Such treatments are beyond the scope of the present discussion.

### ELASTIC DEFORMATION

#### 6.3 Stress–Strain Behavior

The degree to which a structure deforms or strains depends on the magnitude of an imposed stress. For most metals that are stressed in tension and at relatively low levels, stress and strain are proportional to each other through the relationship

\[
\sigma = E \varepsilon \quad (6.5)
\]

This is known as Hooke's law, and the constant of proportionality \( E \) (GPa or psi) is the **modulus of elasticity**, or **Young's modulus**. For most typical metals the magnitude of this modulus ranges between 45 GPa \((6.5 \times 10^6 \text{ psi})\), for magnesium, and 407 GPa \((59 \times 10^6 \text{ psi})\), for tungsten. Modulus of elasticity values for several metals at room temperature are presented in Table 6.1.

Deformation in which stress and strain are proportional is called **elastic deformation**; a plot of stress (ordinate) versus strain (abscissa) results in a linear relation-

<table>
<thead>
<tr>
<th>Metal Alloy</th>
<th>Modulus of Elasticity</th>
<th>Shear Modulus</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPa ( \times 10^6 \text{ psi} )</td>
<td>GPa ( \times 10^6 \text{ psi} )</td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>69 ( \times 10^6 )</td>
<td>25 ( \times 10^6 )</td>
<td>0.33</td>
</tr>
<tr>
<td>Brass</td>
<td>97 ( \times 10^6 )</td>
<td>37 ( \times 10^6 )</td>
<td>0.34</td>
</tr>
<tr>
<td>Copper</td>
<td>110 ( \times 10^6 )</td>
<td>46 ( \times 10^6 )</td>
<td>0.34</td>
</tr>
<tr>
<td>Magnesium</td>
<td>45 ( \times 6.5 )</td>
<td>17 ( \times 2.5 )</td>
<td>0.29</td>
</tr>
<tr>
<td>Nickel</td>
<td>207 ( \times 30 )</td>
<td>76 ( \times 11.0 )</td>
<td>0.31</td>
</tr>
<tr>
<td>Steel</td>
<td>207 ( \times 30 )</td>
<td>83 ( \times 12.0 )</td>
<td>0.30</td>
</tr>
<tr>
<td>Titanium</td>
<td>107 ( \times 15.5 )</td>
<td>45 ( \times 6.5 )</td>
<td>0.34</td>
</tr>
<tr>
<td>Tungsten</td>
<td>407 ( \times 59 )</td>
<td>160 ( \times 23.2 )</td>
<td>0.28</td>
</tr>
</tbody>
</table>

---


6. The SI unit for the modulus of elasticity is gigapascal, GPa, where 1 GPa = \( 10^9 \) N/m\(^2\) = \( 10^3 \) MPa.
6.3 Stress–Strain Behavior

The relationship, as shown in Figure 6.5. The slope of this linear segment corresponds to the modulus of elasticity $E$. This modulus may be thought of as stiffness, or a material's resistance to elastic deformation. The greater the modulus, the stiffer the material, or the smaller the elastic strain that results from the application of a given stress. The modulus is an important design parameter used for computing elastic deflections.

Elastic deformation is nonpermanent, which means that when the applied load is released, the piece returns to its original shape. As shown in the stress–strain plot (Figure 6.5), application of the load corresponds to moving from the origin up and along the straight line. Upon release of the load, the line is traversed in the opposite direction, back to the origin.

There are some materials (e.g., gray cast iron, concrete, and many polymers) for which this initial elastic portion of the stress–strain curve is not linear (Figure 6.6); hence, it is not possible to determine a modulus of elasticity as described above. For this nonlinear behavior, either tangent or secant modulus is normally used. Tangent modulus is taken as the slope of the stress–strain curve at some specified level of stress, while secant modulus represents the slope of a secant drawn from the origin to some given point of the $\sigma$–$\epsilon$ curve. The determination of these moduli is illustrated in Figure 6.6.
On an atomic scale, macroscopic elastic strain is manifested as small changes in the interatomic spacing and the stretching of interatomic bonds. As a consequence, the magnitude of the modulus of elasticity is a measure of the resistance to separation of adjacent atoms, that is, the interatomic bonding forces. Furthermore, this modulus is proportional to the slope of the interatomic force–separation curve (Figure 2.8a) at the equilibrium spacing:

$$E \propto \left(\frac{dF}{dr}\right)_{r_0}$$  \hspace{1cm} (6.6)

Figure 6.7 shows the force–separation curves for materials having both strong and weak interatomic bonds; the slope at $r_0$ is indicated for each.

Values of the modulus of elasticity for ceramic materials are characteristically higher than for metals; for polymers, they are lower. These differences are a direct consequence of the different types of atomic bonding in the three materials types. Furthermore, with increasing temperature, the modulus of elasticity diminishes, as is shown for several metals in Figure 6.8.
As would be expected, the imposition of compressive, shear, or torsional stresses also evokes elastic behavior. The stress–strain characteristics at low stress levels are virtually the same for both tensile and compressive situations, to include the magnitude of the modulus of elasticity. Shear stress and strain are proportional to each other through the expression

\[
\tau = G\gamma
\]  

(6.7)

where \( G \) is the shear modulus, the slope of the linear elastic region of the shear stress–strain curve. Table 6.1 gives the shear moduli for a number of the common metals.

### 6.4 Anelasticity

Up to this point, it has been assumed that elastic deformation is time independent, that is, that an applied stress produces an instantaneous elastic strain that remains constant over the period of time the stress is maintained. It has also been assumed that upon release of the load the strain is totally recovered, that is, that the strain immediately returns to zero. In most engineering materials, however, there will also exist a time-dependent elastic strain component. That is, elastic deformation will continue after the stress application, and upon load release some finite time is required for complete recovery. This time-dependent elastic behavior is known as anelasticity, and it is due to time-dependent microscopic and atomistic processes that are attendant to the deformation. For metals the anelastic component is normally small and is often neglected. However, for some polymeric materials its magnitude is significant; in this case it is termed viscoelastic behavior, which is the discussion topic of Section 16.7.

### Example Problem 6.1

A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?

**Solution**

Since the deformation is elastic, strain is dependent on stress according to Equation 6.5. Furthermore, the elongation \( \Delta l \) is related to the original length \( l_0 \) through Equation 6.2. Combining these two expressions and solving for \( \Delta l \) yields

\[
\sigma = \varepsilon E = \left( \frac{\Delta l}{l_0} \right) E
\]

\[
\Delta l = \frac{\sigma l_0}{E}
\]

The values of \( \sigma \) and \( l_0 \) are given as 276 MPa and 305 mm, respectively, and the magnitude of \( E \) for copper from Table 6.1 is 110 GPa (16 × 10^6 psi). Elongation is obtained by substitution into the expression above as

\[
\Delta l = \frac{(276 \text{ MPa})(305 \text{ mm})}{110 \times 10^9 \text{ MPa}} = 0.77 \text{ mm (0.03 in.)}
\]
6.5 Elastic Properties of Materials

When a tensile stress is imposed on a metal specimen, an elastic elongation and accompanying strain $\epsilon_z$ result in the direction of the applied stress (arbitrarily taken to be the $z$ direction), as indicated in Figure 6.9. As a result of this elongation, there will be contractions in the lateral ($x$ and $y$) directions perpendicular to the applied stress; from these contractions, the compressive strains $\epsilon_x$ and $\epsilon_y$ may be determined. If the applied stress is uniaxial (only in the $z$ direction), and the material is isotropic, then $\epsilon_x = \epsilon_y$. A parameter termed Poisson’s ratio $\nu$ is defined as the ratio of the lateral and axial strains, or

$$
\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z} \tag{6.8}
$$

The negative sign is included in the expression so that $\nu$ will always be positive, since $\epsilon_x$ and $\epsilon_y$ will always be of opposite sign. Theoretically, Poisson’s ratio for isotropic materials should be $\frac{1}{2}$; furthermore, the maximum value for $\nu$ (or that value for which there is no net volume change) is 0.50. For many metals and other alloys, values of Poisson’s ratio range between 0.25 and 0.35. Table 6.1 shows $\nu$ values for several common metallic materials.

For isotropic materials, shear and elastic moduli are related to each other and to Poisson’s ratio according to

$$
E = 2G(1 + \nu) \tag{6.9}
$$

In most metals $G$ is about $0.4E$; thus, if the value of one modulus is known, the other may be approximated.

Many materials are elastically anisotropic; that is, the elastic behavior (e.g., the magnitude of $E$) varies with crystallographic direction (see Table 3.3). For these materials the elastic properties are completely characterized only by the specification of several elastic constants, their number depending on characteristics of the crystal structure. Even for isotropic materials, for complete characterization of the elastic

**Figure 6.9** Axial ($z$) elongation (positive strain) and lateral ($x$ and $y$) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.
properties, at least two constants must be given. Since the grain orientation is random in most polycrystalline materials, these may be considered to be isotropic; inorganic ceramic glasses are also isotropic. The remaining discussion of mechanical behavior assumes isotropy and polycrystallinity because such is the character of most engineering materials.

**Example Problem 6.2**

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a $2.5 \times 10^{-3}$ mm ($10^{-4}$ in.) change in diameter if the deformation is entirely elastic.

**Solution**

This deformation situation is represented in the accompanying drawing.

When the force $F$ is applied, the specimen will elongate in the $z$ direction and at the same time experience a reduction in diameter. $\Delta d$, of $2.5 \times 10^{-3}$ mm in the $x$ direction. For the strain in the $x$ direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3} \text{ mm}}{10 \text{ mm}} = -2.5 \times 10^{-4}$$

which is negative, since the diameter is reduced.

It next becomes necessary to calculate the strain in the $z$ direction using Equation 6.8. The value for Poisson’s ratio for brass is 0.34 (Table 6.1), and thus

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{-2.5 \times 10^{-4}}{0.34} = 7.35 \times 10^{-4}$$

The applied stress may now be computed using Equation 6.5 and the modulus of elasticity, given in Table 6.1 as 97 GPa ($14 \times 10^6$ psi), as

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^9 \text{ MPa}) = 71.3 \text{ MPa}$$
Finally, from Equation 6.1, the applied force may be determined as

\[ F = \sigma A_0 = \sigma \left( \frac{d_0}{2} \right)^2 \pi = (71.3 \times 10^6 \text{N/m}^2) \left( \frac{10 \times 10^{-3} \text{m}}{2} \right)^2 \pi = 5600 \text{N} \ (1293 \text{ lb}) \]

**PLASTIC DEFORMATION**

For most metallic materials, elastic deformation persists only to strains of about 0.005. As the material is deformed beyond this point, the stress is no longer proportional to strain (Hooke’s law, Equation 6.5, ceases to be valid), and permanent, nonreversible, or plastic deformation occurs. Figure 6.10a plots schematically the tensile stress–strain behavior into the plastic region for a typical metal. The transition from elastic to plastic is a gradual one for most metals; some curvature results at the onset of plastic deformation, which increases more rapidly with rising stress.

From an atomic perspective, plastic deformation corresponds to the breaking of bonds with original atom neighbors and then reforming bonds with new neighbors as large numbers of atoms or molecules move relative to one another; upon removal of the stress they do not return to their original positions. The mechanism of this deformation is different for crystalline and amorphous materials. For crystalline solids, deformation is accomplished by means of a process called slip, which involves the motion of dislocations as discussed in Section 7.2. Plastic deformation in noncrystalline solids (as well as liquids) occurs by a viscous flow mechanism, which is outlined in Section 13.9.
6.6 TENSILE PROPERTIES

YIELDING AND YIELD STRENGTH

Most structures are designed to ensure that only elastic deformation will result when a stress is applied. It is therefore desirable to know the stress level at which plastic deformation begins, or where the phenomenon of yielding occurs. For metals that experience this gradual elastic–plastic transition, the point of yielding may be determined as the initial departure from linearity of the stress–strain curve; this is sometimes called the proportional limit, as indicated by point \( P \) in Figure 6.10a. In such cases the position of this point may not be determined precisely. As a consequence, a convention has been established wherein a straight line is constructed parallel to the elastic portion of the stress–strain curve at some specified strain offset, usually 0.002. The stress corresponding to the intersection of this line and the stress–strain curve as it bends over in the plastic region is defined as the yield strength \( \sigma_y \). This is demonstrated in Figure 6.10a. Of course, the units of yield strength are MPa or psi.

For those materials having a nonlinear elastic region (Figure 6.6), use of the strain offset method is not possible, and the usual practice is to define the yield strength as the stress required to produce some amount of strain (e.g., \( \epsilon = 0.005 \)).

Some steels and other materials exhibit the tensile stress–strain behavior as shown in Figure 6.10b. The elastic–plastic transition is very well defined and occurs abruptly in what is termed a yield point phenomenon. At the upper yield point, plastic deformation is initiated with an actual decrease in stress. Continued deformation fluctuates slightly about some constant stress value, termed the lower yield point: stress subsequently rises with increasing strain. For metals that display this effect, the yield strength is taken as the average stress that is associated with the lower yield point, since it is well defined and relatively insensitive to the testing procedure. Thus, it is not necessary to employ the strain offset method for these materials.

The magnitude of the yield strength for a metal is a measure of its resistance to plastic deformation. Yield strengths may range from 35 MPa (5000 psi) for a low-strength aluminum to over 1400 MPa (200,000 psi) for high-strength steels.

TENSILE STRENGTH

After yielding, the stress necessary to continue plastic deformation in metals increases to a maximum, point \( M \) in Figure 6.11, and then decreases to the eventual fracture, point \( F \). The tensile strength \( TS \) (MPa or psi) is the stress at the maximum on the engineering stress–strain curve (Figure 6.11). This corresponds to the maximum stress that can be sustained by a structure in tension; if this stress is applied and maintained, fracture will result. All deformation up to this point is uniform throughout the narrow region of the tensile specimen. However, at this maximum stress, a small constriction or neck begins to form at some point, and all subsequent deformation is confined at this neck, as indicated by the schematic specimen insets.

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7 "Strength" is used in lieu of "stress" because strength is a property of the metal, whereas stress is related to the magnitude of the applied load.
8 For customary U.S. units, the unit of kilopounds per square inch (ksi) is sometimes used for the sake of convenience, where

\[ 1 \text{ ksi} = 1000 \text{ psi} \]
9 It should be pointed out that to observe the yield point phenomenon, a "stiff" tensile-testing apparatus must be used; by stiff is meant that there is very little elastic deformation of the machine during loading.
in Figure 6.11. This phenomenon is termed “necking,” and fracture ultimately occurs at the neck. The fracture strength corresponds to the stress at fracture.

Tensile strengths may vary anywhere from 50 MPa (7000 psi) for an aluminum to as high as 3000 MPa (450,000 psi) for the high-strength steels. Ordinarily, when the strength of a metal is cited for design purposes, the yield strength is used. This is because by the time a stress corresponding to the tensile strength has been applied, often a structure has experienced so much plastic deformation that it is useless. Furthermore, fracture strengths are not normally specified for engineering design purposes.

**Example Problem 6.3**

From the tensile stress–strain behavior for the brass specimen shown in Figure 6.12, determine the following:

(a) The modulus of elasticity.
(b) The yield strength at a strain offset of 0.002.
(c) The maximum load that can be sustained by a cylindrical specimen having an original diameter of 12.8 mm (0.505 in.).
(d) The change in length of a specimen originally 250 mm (10 in.) long that is subjected to a tensile stress of 345 MPa (50,000 psi).

**Solution**

(a) The modulus of elasticity is the slope of the elastic or initial linear portion of the stress–strain curve. The strain axis has been expanded in the inset, Figure 6.12, to facilitate this computation. The slope of this linear region is the rise
over the run, or the change in stress divided by the corresponding change in strain; in mathematical terms,

\[ E = \text{slope} = \frac{\Delta \sigma}{\Delta \varepsilon} = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} \] (6.10)

Inasmuch as the line segment passes through the origin, it is convenient to take both \( \sigma_1 \) and \( \varepsilon_1 \) as zero. If \( \sigma_2 \) is arbitrarily taken as 150 MPa, then \( \varepsilon_2 \) will have a value of 0.0016. Therefore,

\[ E = \frac{(150 - 0) \text{ MPa}}{0.0016 - 0} = 93.8 \text{ GPa} \left( 13.6 \times 10^6 \text{ psi} \right) \]

which is very close to the value of 97 GPa \((14 \times 10^6 \text{ psi})\) given for brass in Table 6.1.

(b) The 0.002 strain offset line is constructed as shown in the inset; its intersection with the stress–strain curve is at approximately 250 MPa \((36,000 \text{ psi})\), which is the yield strength of the brass.

(c) The maximum load that can be sustained by the specimen is calculated by using Equation 6.1, in which \( \sigma \) is taken to be the tensile strength, from Figure 6.12. 450 MPa \((65,000 \text{ psi})\). Solving for \( F \), the maximum load, yields

\[ F = \sigma A_0 = \sigma \left( \frac{d_0}{2} \right)^2 \pi \]

\[ = (450 \times 10^6 \text{ N/m}^2) \left( \frac{12.8 \times 10^{-3} \text{ m}}{2} \right)^2 \pi = 57,900 \text{ N} \left( 13,000 \text{ lb} \right) \]

(d) To compute the change in length, \( \Delta l \), in Equation 6.2, it is first necessary to determine the strain that is produced by a stress of 345 MPa. This is accom-
plished by locating the stress point on the stress–strain curve, point A, and reading the corresponding strain from the strain axis, which is approximately 0.06. Inasmuch as \( l_0 = 250 \text{ mm} \), we have

\[
\Delta l = \varepsilon l_0 = (0.06)(250 \text{ mm}) = 15 \text{ mm (0.6 in.)}
\]

**DUCTILITY**

**Ductility** is another important mechanical property. It is a measure of the degree of plastic deformation that has been sustained at fracture. A material that experiences very little or no plastic deformation upon fracture is termed **brittle**. The tensile stress–strain behaviors for both ductile and brittle materials are schematically illustrated in Figure 6.13.

Ductility may be expressed quantitatively as either **percent elongation** or **percent reduction in area**. The percent elongation \( \%\text{EL} \) is the percentage of plastic strain at fracture, or

\[
\%\text{EL} = \left( \frac{l_f - l_0}{l_0} \right) \times 100
\]  

(6.11)

where \( l_f \) is the fracture length\(^{10} \) and \( l_0 \) is the original gauge length as above. Inasmuch as a significant proportion of the plastic deformation at fracture is confined to the neck region, the magnitude of \( \%\text{EL} \) will depend on specimen gauge length. The shorter \( l_0 \), the greater is the fraction of total elongation from the neck and, consequently, the higher the value of \( \%\text{EL} \). Therefore, \( l_0 \) should be specified when percent elongation values are cited; it is commonly 50 mm (2 in.).

Percent reduction in area \( \%\text{RA} \) is defined as

\[
\%\text{RA} = \left( \frac{A_0 - A_f}{A_0} \right) \times 100
\]

(6.12)

where \( A_0 \) is the original cross-sectional area and \( A_f \) is the cross-sectional area at the point of fracture.\(^{10} \) Percent reduction in area values are independent of both

\(^{10} \) Both \( l_0 \) and \( A_0 \) are measured subsequent to fracture, and after the two broken ends have been repositioned back together.
Table 6.2  Typical Mechanical Properties of Several Metals and Alloys in an Annealed State

<table>
<thead>
<tr>
<th>Metal Alloy</th>
<th>Yield Strength MPa (ksi)</th>
<th>Tensile Strength MPa (ksi)</th>
<th>Ductility, %EL [in 50 mm (2 in.)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>35 (5)</td>
<td>90 (13)</td>
<td>40</td>
</tr>
<tr>
<td>Copper</td>
<td>69 (10)</td>
<td>200 (29)</td>
<td>45</td>
</tr>
<tr>
<td>Brass (70Cu–30Zn)</td>
<td>75 (11)</td>
<td>300 (44)</td>
<td>68</td>
</tr>
<tr>
<td>Iron</td>
<td>130 (19)</td>
<td>262 (38)</td>
<td>45</td>
</tr>
<tr>
<td>Nickel</td>
<td>138 (20)</td>
<td>480 (70)</td>
<td>40</td>
</tr>
<tr>
<td>Steel (1020)</td>
<td>180 (26)</td>
<td>380 (55)</td>
<td>25</td>
</tr>
<tr>
<td>Titanium</td>
<td>450 (65)</td>
<td>520 (75)</td>
<td>25</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>565 (82)</td>
<td>655 (95)</td>
<td>35</td>
</tr>
</tbody>
</table>

$l_0$ and $A_0$. Furthermore, for a given material the magnitudes of %EL and %RA will, in general, be different. Most metals possess at least a moderate degree of ductility at room temperature; however, some become brittle as the temperature is lowered (Section 8.6).

A knowledge of the ductility of materials is important for at least two reasons. First, it indicates to the designer the degree to which a structure will deform plastically before fracture. Second, it specifies the degree of allowable deformation during fabrication operations. We sometimes refer to relatively ductile materials as being "forgiving," in the sense that they may experience local deformation without fracture should there be an error in the magnitude of the design stress calculation.

Brittle materials are approximately considered to be those having a fracture strain of less than about 5%.

Thus, several important mechanical properties of metals may be determined from tensile stress–strain tests. Table 6.2 presents some typical room-temperature values of yield strength, tensile strength, and ductility for several of the common metals. These properties are sensitive to any prior deformation, the presence of impurities, and/or any heat treatment to which the metal has been subjected. The modulus of elasticity is one mechanical parameter that is insensitive to these treatments. As with modulus of elasticity, the magnitudes of both yield and tensile strengths decline with increasing temperature; just the reverse holds for ductility—it usually increases with temperature. Figure 6.14 shows how the stress–strain behavior of iron varies with temperature.

![Figure 6.14](image-url) Engineering stress–strain behavior for iron at three temperatures.
RESILIENCE

Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered. The associated property is the *modulus of resilience*, $U_r$, which is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding.

Computationally, the modulus of resilience for a specimen subjected to a uni-axial tension test is just the area under the engineering stress–strain curve taken to yielding (Figure 6.15), or

$$U_r = \int_0^{\varepsilon_y} \sigma \, d\varepsilon$$

(6.13a)

Assuming a linear elastic region,

$$U_r = \frac{1}{2} \sigma_y \varepsilon_y$$

(6.13b)

in which $\varepsilon_y$ is the strain at yielding.

The units of resilience are the product of the units from each of the two axes of the stress–strain plot. For SI units, this is joules per cubic meter ($J/m^3$, equivalent to Pa), whereas with Customary U.S. units it is inch-pounds force per cubic inch (in.-lb/in.$^3$, equivalent to psi). Both joules and inch-pounds force are units of energy, and thus this area under the stress–strain curve represents energy absorption per unit volume (in cubic meters or cubic inches) of material.

Incorporation of Equation 6.5 into Equation 6.13b yields

$$U_r = \frac{1}{2} \sigma_y \varepsilon_y = \frac{1}{2} \sigma_y \left( \frac{\sigma_y}{E} \right) = \frac{\sigma_y^2}{2E}$$

(6.14)

Thus, resilient materials are those having high yield strengths and low moduli of elasticity; such alloys would be used in spring applications.

TOUGHNESS

Toughness is a mechanical term that is used in several contexts; loosely speaking, it is a measure of the ability of a material to absorb energy up to fracture. Specimen geometry as well as the manner of load application are important in toughness.

**Figure 6.15** Schematic representation showing how modulus of resilience (corresponding to the shaded area) is determined from the tensile stress–strain behavior of a material.
determinations. For dynamic (high strain rate) loading conditions and when a notch (or point of stress concentration) is present, notch toughness is assessed by using an impact test, as discussed in Section 8.6. Furthermore, fracture toughness is a property indicative of a material’s resistance to fracture when a crack is present (Section 8.5).

For the static (low strain rate) situation, toughness may be ascertained from the results of a tensile stress–strain test. It is the area under the $\sigma$–$\varepsilon$ curve up to the point of fracture. The units for toughness are the same as for resilience (i.e., energy per unit volume of material). For a material to be tough, it must display both strength and ductility; and often, ductile materials are tougher than brittle ones. This is demonstrated in Figure 6.13, in which the stress–strain curves are plotted for both material types. Hence, even though the brittle material has higher yield and tensile strengths, it has a lower toughness than the ductile one, by virtue of lack of ductility; this is deduced by comparing the areas $ABC$ and $AB'C'$ in Figure 6.13.

### 6.7 True Stress and Strain

From Figure 6.11, the decline in the stress necessary to continue deformation past the maximum point $M$, seems to indicate that the material is becoming weaker. This is not at all the case: as a matter of fact, it is increasing in strength. However, the cross-sectional area is decreasing rapidly within the neck region, where deformation is occurring. This results in a reduction in the load-bearing capacity of the specimen. The stress, as computed from Equation 6.1, is on the basis of the original cross-sectional area before any deformation, and does not take into account this diminution in area at the neck.

Sometimes it is more meaningful to use a true stress–true strain scheme. True stress $\sigma_T$ is defined as the load $F$ divided by the instantaneous cross-sectional area $A_I$ over which deformation is occurring (i.e., the neck, past the tensile point), or

\[
\sigma_T = \frac{F}{A_I} \quad (6.15)
\]

Furthermore, it is occasionally more convenient to represent strain as true strain $\varepsilon_T$, defined by

\[
\varepsilon_T = \ln \frac{l}{l_0} \quad (6.16)
\]

If no volume change occurs during deformation, that is, if

\[
A_I l_I = A_0 l_0 \quad (6.17)
\]

true and engineering stress and strain are related according to

\[
\sigma_T = \sigma (1 + \varepsilon) \quad (6.18a)
\]

\[
\varepsilon_T = \ln(1 + \varepsilon) \quad (6.18b)
\]

Equations 6.18a and 6.18b are valid only to the onset of necking; beyond this point true stress and strain should be computed from actual load, cross-sectional area, and gauge length measurements.
A schematic comparison of engineering and true stress–strain behavior is made in Figure 6.16. It is worth noting that the true stress necessary to sustain increasing strain continues to rise past the tensile point $M'$.

Coincident with the formation of a neck is the introduction of a complex stress state within the neck region (i.e., the existence of other stress components in addition to the axial stress). As a consequence, the correct stress (axial) within the neck is slightly lower than the stress computed from the applied load and neck cross-sectional area. This leads to the “corrected” curve in Figure 6.16.

For some metals and alloys the region of the true stress-strain curve from the onset of plastic deformation to the point at which necking begins may be approximated by

$$\sigma_T = Kn^p$$  \hspace{1cm} (6.19)

In this expression, $K$ and $n$ are constants, which values will vary from alloy to alloy, and will also depend on the condition of the material (i.e., whether it has been plastically deformed, heat treated, etc.). The parameter $n$ is often termed the strain-hardening exponent and has a value less than unity. Values of $n$ and $K$ for several alloys are contained in Table 6.3.

<table>
<thead>
<tr>
<th>Material</th>
<th>$n$</th>
<th>MPa</th>
<th>psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-carbon steel</td>
<td>0.26</td>
<td>530</td>
<td>77,000</td>
</tr>
<tr>
<td>(annealed)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alloy steel</td>
<td>0.15</td>
<td>640</td>
<td>93,000</td>
</tr>
<tr>
<td>(Type 4340, annealed)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stainless steel</td>
<td>0.45</td>
<td>1275</td>
<td>185,000</td>
</tr>
<tr>
<td>(Type 304, annealed)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminum (annealed)</td>
<td>0.20</td>
<td>180</td>
<td>26,000</td>
</tr>
<tr>
<td>Aluminum alloy</td>
<td>0.16</td>
<td>690</td>
<td>100,000</td>
</tr>
<tr>
<td>(Type 2024, heat treated)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copper (annealed)</td>
<td>0.54</td>
<td>315</td>
<td>46,000</td>
</tr>
<tr>
<td>Brass</td>
<td>0.49</td>
<td>895</td>
<td>130,000</td>
</tr>
<tr>
<td>(70Cu–30Zn, annealed)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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EXAMPLE PROBLEM 6.4

A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile tested to fracture and found to have an engineering fracture strength $\sigma_f$ of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine:

(a) The ductility in terms of percent reduction in area.
(b) The true stress at fracture.

**Solution**

(a) Ductility is computed using Equation 6.12, as

$$\% RA = \left( \frac{\left( \frac{12.8 \text{ mm}}{2} \right)^2 \pi - \left( \frac{10.7 \text{ mm}}{2} \right)^2 \pi}{\left( \frac{12.8 \text{ mm}}{2} \right)^2 \pi} \right) \times 100$$

$$= \frac{128.7 \text{ mm}^2 - 89.9 \text{ mm}^2}{128.7 \text{ mm}^2} \times 100 = 30\%$$

(b) True stress is defined by Equation 6.15, where in this case the area is taken as the fracture area $A_f$. However, the load at fracture must first be computed from the fracture strength as

$$F = \sigma_f A_0 = (460 \times 10^6 \text{ N/m}^2)(128.7 \text{ mm}^2) \left( \frac{1 \text{ m}^2}{10^6 \text{ mm}^2} \right) = 59,200 \text{ N}$$

Thus, the true stress is calculated as

$$\sigma_f = \frac{F}{A_f} = \frac{59,200 \text{ N}}{(89.9 \text{ mm}^2) \left( \frac{1 \text{ m}^2}{10^6 \text{ mm}^2} \right)}$$

$$= 6.6 \times 10^6 \text{ N/m}^2 = 660 \text{ MPa (95,700 psi)}$$

EXAMPLE PROBLEM 6.5

Compute the strain-hardening exponent $n$ in Equation 6.19 for an alloy in which a true stress of 415 MPa (60,000 psi) produces a true strain of 0.10; assume a value of 1035 MPa (150,000 psi) for $K$.

**Solution**

This requires some algebraic manipulation of Equation 6.19 so that $n$ becomes the dependent parameter. This is accomplished by taking logarithms and rearranging. Solving for $n$ yields

$$n = \frac{\log \sigma_f - \log K}{\log \varepsilon_f}$$

$$= \frac{\log(415 \text{ MPa}) - \log(1035 \text{ MPa})}{\log(0.1)} = 0.40$$
6.8 **Elastic Recovery During Plastic Deformation**

Upon release of the load during the course of a stress–strain test, some fraction of the total deformation is recovered as elastic strain. This behavior is demonstrated in Figure 6.17, a schematic engineering stress–strain plot. During the unloading cycle, the curve traces a near straight-line path from the point of unloading (point $D$), and its slope is virtually identical to the modulus of elasticity, or parallel to the initial elastic portion of the curve. The magnitude of this elastic strain, which is regained during unloading, corresponds to the strain recovery, as shown in Figure 6.17. If the load is reapplied, the curve will traverse essentially the same linear portion in the direction opposite to unloading; yielding will again occur at the unloading stress level where the unloading began. There will also be an elastic strain recovery associated with fracture.

6.9 **Compressive, Shear, and Torsional Deformation**

Of course, metals may experience plastic deformation under the influence of applied compressive, shear, and torsional loads. The resulting stress–strain behavior into the plastic region will be similar to the tensile counterpart (Figure 6.10a: yielding and the associated curvature). However, for compression, there will be no maximum, since necking does not occur; furthermore, the mode of fracture will be different from that for tension.

6.10 **Hardness**

Another mechanical property that may be important to consider is hardness, which is a measure of a material's resistance to localized plastic deformation (e.g., a small dent or a scratch). Early hardness tests were based on natural minerals with a scale constructed solely on the ability of one material to scratch another that was softer.

**Figure 6.17** Schematic tensile stress–strain diagram showing the phenomena of elastic strain recovery and strain hardening. The initial yield strength is designated as $\sigma_y$; $\sigma_y$ is the yield strength after releasing the load at point $D$, and then upon reloading.
As a result of uncertainties in both measured mechanical properties and inservice applied stresses, design or safe stresses are normally utilized for design purposes. For ductile materials, safe stress is the ratio of the yield strength and the factor of safety.

**IMPORTANT TERMS AND CONCEPTS**

<table>
<thead>
<tr>
<th>Anelasticity</th>
<th>Hardness</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design stress</td>
<td>Modulus of elasticity</td>
<td>Tensile strength</td>
</tr>
<tr>
<td>Ductility</td>
<td>Plastic deformation</td>
<td>Toughness</td>
</tr>
<tr>
<td>Elastic deformation</td>
<td>Poisson’s ratio</td>
<td>True strain</td>
</tr>
<tr>
<td>Elastic recovery</td>
<td>Proportional limit</td>
<td>True stress</td>
</tr>
<tr>
<td>Engineering strain</td>
<td>Resilience</td>
<td>Yielding</td>
</tr>
<tr>
<td>Engineering stress</td>
<td>Safe stress</td>
<td>Yield strength</td>
</tr>
</tbody>
</table>

**REFERENCES**


**QUESTIONS AND PROBLEMS**

6.1 Using mechanics of materials principles (i.e., equations of mechanical equilibrium applied to a free-body diagram), derive Equations 6.4a and 6.4b.

6.2 (a) Equations 6.4a and 6.4b are expressions for normal ($\sigma'$) and shear ($\tau'$) stresses, respectively, as a function of the applied tensile stress ($\sigma$) and the inclination angle of the plane on which these stresses are taken ($\theta$ of Figure 6.4). Make a plot on which is presented the orientation parameters of these expressions (i.e., $\cos^2 \theta$ and $\sin \theta \cos \theta$) versus $\theta$.

(b) From this plot, at what angle of inclination is the normal stress a maximum?

(c) Also, at what inclination angle is the shear stress a maximum?

6.3 A specimen of aluminum having a rectangular cross section 10 mm $\times$ 12.7 mm (0.4 in. $\times$ 0.5 in.) is pulled in tension with 35,500 N (8000 lb$_f$) force, producing only elastic deformation. Calculate the resulting strain.

6.4 A cylindrical specimen of a titanium alloy having an elastic modulus of 107 GPa (15.5 $\times$ 10$^6$ psi) and an original diameter of 3.8 mm (0.15 in.) will experience only elastic deformation when a tensile load of 2000 N (450 lb$_f$) is applied. Compute the maximum length of the specimen before deformation if the maximum allowable elongation is 0.42 mm (0.0165 in.).

6.5 A steel bar 100 mm (4.0 in.) long and having a square cross section 20 mm (0.8 in.) on an edge is pulled in tension with a load of 89,000 N (20,000 lb$_f$), and experiences an elongation of 0.10 mm (4.0 $\times$ 10$^{-3}$ in.). Assuming that the deformation is entirely elastic, calculate the elastic modulus of the steel.
6.6 Consider a cylindrical titanium wire 3.0 mm (0.12 in.) in diameter and $2.5 \times 10^4$ mm (1000 in.) long. Calculate its elongation when a load of 500 N (112 lb.) is applied. Assume that the deformation is totally elastic.

6.7 For a bronze alloy, the stress at which plastic deformation begins is 275 MPa (40,000 psi), and the modulus of elasticity is 115 GPa (16.7 $\times 10^9$ psi).

(a) What is the maximum load that may be applied to a specimen with a cross-sectional area of 325 mm$^2$ (0.5 in.$^2$) without plastic deformation?

(b) If the original specimen length is 115 mm (4.5 in.), what is the maximum length to which it may be stretched without causing plastic deformation?

6.8 A cylindrical rod of copper ($E = 110$ GPa, $16 \times 10^6$ psi) having a yield strength of 240 MPa (35,000 psi) is to be subjected to a load of 6660 N (1500 lb.). If the length of the rod is 380 mm (15.0 in.), what must be the diameter to allow an elongation of 0.50 mm (0.020 in.)?

6.9 Consider a cylindrical specimen of a steel alloy (Figure 6.21) 10 mm (0.39 in.) in diameter and 75 mm (3.0 in.) long that is pulled in tension. Determine its elongation when a load of 23,500 N (5300 lb.) is applied.

6.10 Figure 6.22 shows, for a gray cast iron, the tensile engineering stress–strain curve in the elastic region. Determine (a) the secant modulus taken to 35 MPa (5000 psi), and (b) the tangent modulus taken from the origin.
6.11 As was noted in Section 3.14, for single crystals of some substances, the physical properties are anisotropic, that is, they are dependent on crystallographic direction. One such property is the modulus of elasticity. For cubic single crystals, the modulus of elasticity in a general [uvw] direction, $E_{uvw}$, is described by the relationship

$$
\frac{1}{E_{uvw}} = \frac{1}{E_{(100)}} - 3 \left( \frac{1}{E_{(100)}} - \frac{1}{E_{(111)}} \right) \left( \alpha^2 \beta^2 + \beta^2 \gamma^2 + \gamma^2 \alpha^2 \right)
$$

where $E_{(100)}$ and $E_{(111)}$ are the moduli of elasticity in [100] and [111] directions, respectively; $\alpha$, $\beta$, and $\gamma$ are the cosines of the angles between [uvw] and the respective [100], [010], and [001] directions. Verify that the $E_{(110)}$ values for aluminum, copper, and iron in Table 3.3 are correct.

6.12 In Section 2.6 it was noted that the net bonding energy $E_N$ between two isolated positive and negative ions is a function of interionic distance $r$ as follows:

$$
E_N = -\frac{A}{r} + \frac{B}{r^n} \quad (6.25)
$$

where $A$, $B$, and $n$ are constants for the particular ion pair. Equation 6.25 is also valid for the bonding energy between adjacent ions in solid materials. The modulus of elasticity $E$ is proportional to the slope of the interionic force-separation curve at the equilibrium interionic separation; that is,

$$
E \propto \frac{dF}{dr}
$$

Derive an expression for the dependence of the modulus of elasticity on these $A$, $B$, and $n$ parameters (for the two-ion system) using the following procedure:

1. Establish a relationship for the force $F$ as a function of $r$, realizing that

$$
F = \frac{dE_N}{dr}
$$

2. Now take the derivative $dF/dr$.

3. Develop an expression for $r_0$, the equilibrium separation. Since $r_0$ corresponds to the value of $r$ at the minimum of the $E_N$-versus-$r$-curve (Figure 2.8b), take the derivative $dE_N/dr$, set it equal to zero, and solve for $r$, which corresponds to $r_0$.

4. Finally, substitute this expression for $r_0$ into the relationship obtained by taking $dF/dr$.

6.13 Using the solution to Problem 6.12, rank the magnitudes of the moduli of elasticity for the following hypothetical X, Y, and Z materials from the greatest to the least. The appropriate $A$, $B$, and $n$ parameters (Equation 6.25) for these three materials are tabulated below: they yield $E_N$ in units of electron volts and $r$ in nanometers:

<table>
<thead>
<tr>
<th>Material</th>
<th>$A$</th>
<th>$B$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>2.5</td>
<td>$2 \times 10^{-5}$</td>
<td>8</td>
</tr>
<tr>
<td>Y</td>
<td>2.3</td>
<td>$8 \times 10^{-6}$</td>
<td>10.5</td>
</tr>
<tr>
<td>Z</td>
<td>3.0</td>
<td>$1.5 \times 10^{-5}$</td>
<td>9</td>
</tr>
</tbody>
</table>

6.14 A cylindrical specimen of aluminum having a diameter of 19 mm (0.75 in.) and length of 200 mm (8.0 in.) is deformed elastically in tension with a force of 48,800 N (11,000 lb.). Using the data contained in Table 6.1, determine the following:

(a) The amount by which this specimen will elongate in the direction of the applied stress.

(b) The change in diameter of the specimen. Will the diameter increase or decrease?

6.15 A cylindrical bar of steel 10 mm (0.4 in.) in diameter is to be deformed elastically by application of a force along the bar axis. Using the data in Table 6.1, determine the force that will produce an elastic reduction of $3 \times 10^{-3}$ mm ($1.2 \times 10^{-4}$ in.) in the diameter.

6.16 A cylindrical specimen of some alloy 8 mm (0.31 in.) in diameter is stressed elastically in tension. A force of 15,700 N (3530 lb.) produces a reduction in specimen diameter of $5 \times 10^{-4}$ mm ($2 \times 10^{-4}$ in.). Compute Poisson’s ratio for this material if its modulus of elasticity is 140 GPa ($20.3 \times 10^6$ psi).

6.17 A cylindrical specimen of a hypothetical metal alloy is stressed in compression. If its original and final diameters are 20.000 and 20.025 mm, respectively, and its final length is 74.96 mm, compute its original length if the deformation is totally elastic. The elastic and shear moduli for this alloy are 105 GPa and 39.7 GPa, respectively.
6.18 Consider a cylindrical specimen of some hypothetical metal alloy that has a diameter of 8.0 mm (0.31 in.). A tensile force of 1000 N (225 lbf) produces an elastic reduction in diameter of $2.8 \times 10^{-4}$ mm ($1.10 \times 10^{-2}$ in.). Compute the modulus of elasticity for this alloy, given that Poisson’s ratio is 0.30.

6.19 A brass alloy is known to have a yield strength of 275 MPa (40,000 psi), a tensile strength of 380 MPa (55,000 psi), and an elastic modulus of 103 GPa ($15.0 \times 10^6$ psi). A cylindrical specimen of this alloy 12.7 mm (0.50 in.) in diameter and 250 mm (10.0 in.) long is stressed in tension and found to elongate 7.6 mm (0.30 in.). On the basis of the information given, is it possible to compute the magnitude of the load that is necessary to produce this change in length? If so, calculate the load. If not, explain why.

6.20 A cylindrical metal specimen 15.0 mm (0.59 in.) in diameter and 150 mm (5.9 in.) long is to be subjected to a tensile stress of 50 MPa (7250 psi); at this stress level the resulting deformation will be totally elastic.

(a) If the elongation must be less than 0.072 mm ($2.83 \times 10^{-3}$ in.), which of the metals in Table 6.1 are suitable candidates? Why?

(b) If, in addition, the maximum permissible diameter decrease is $2.3 \times 10^{-3}$ mm ($9.1 \times 10^{-5}$ in.), which of the metals in Table 6.1 may be used? Why?

6.21 Consider the brass alloy with stress–strain behavior shown in Figure 6.12. A cylindrical specimen of this material 6 mm (0.24 in.) in diameter and 50 mm (2 in.) long is pulled in tension with a force of 5000 N (1125 lbf). If it is known that this alloy has a Poisson’s ratio of 0.30, compute: (a) the specimen elongation, and (b) the reduction in specimen diameter.

6.22 Cite the primary differences between elastic, anelastic, and plastic deformation behaviors.

6.23 A cylindrical rod 100 mm long and having a diameter of 10.0 mm is to be deformed using a tensile load of 27,500 N. It must not experience either plastic deformation or a diameter reduction of more than $7.5 \times 10^{-3}$ mm. Of the materials listed as follows, which are possible candidates? Justify your choice(s).

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Yield Strength (MPa)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum alloy</td>
<td>70</td>
<td>200</td>
<td>0.33</td>
</tr>
<tr>
<td>Brass alloy</td>
<td>101</td>
<td>300</td>
<td>0.35</td>
</tr>
<tr>
<td>Steel alloy</td>
<td>207</td>
<td>400</td>
<td>0.27</td>
</tr>
<tr>
<td>Titanium alloy</td>
<td>107</td>
<td>650</td>
<td>0.36</td>
</tr>
</tbody>
</table>

6.24 A cylindrical rod 380 mm (15.0 in.) long, having a diameter of 10.0 mm (0.40 in.), is to be subjected to a tensile load. If the rod is to experience neither plastic deformation nor an elongation of more than 0.9 mm (0.035 in.) when the applied load is 24,500 N (5500 lbf), which of the four metals or alloys listed below are possible candidates? Justify your choice(s).

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Yield Strength (MPa)</th>
<th>Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum alloy</td>
<td>70</td>
<td>255</td>
<td>420</td>
</tr>
<tr>
<td>Brass alloy</td>
<td>100</td>
<td>345</td>
<td>420</td>
</tr>
<tr>
<td>Copper</td>
<td>110</td>
<td>250</td>
<td>290</td>
</tr>
<tr>
<td>Steel alloy</td>
<td>207</td>
<td>450</td>
<td>550</td>
</tr>
</tbody>
</table>

6.25 Figure 6.21 shows the tensile engineering stress–strain behavior for a steel alloy.

(a) What is the modulus of elasticity?

(b) What is the proportional limit?

(c) What is the yield strength at a strain offset of 0.002?

(d) What is the tensile strength?

6.26 A cylindrical specimen of a brass alloy having a length of 60 mm (2.36 in.) must elongate only 10.8 mm (0.425 in.) when a tensile load of 50,000 N (11,240 lbf) is applied. Under these circumstances, what must be the radius of the specimen? Consider this brass alloy to have the stress–strain behavior shown in Figure 6.12.

6.27 A load of 44,500 N (10,000 lbf) is applied to a cylindrical specimen of steel (displaying the stress–strain behavior shown in Figure 6.21) that has a cross-sectional diameter of 10 mm (0.40 in.).
(a) Will the specimen experience elastic or plastic deformation? Why?

(b) If the original specimen length is 500 mm (20 in.), how much will it increase in length when this load is applied?

6.28 A bar of a steel alloy that exhibits the stress–strain behavior shown in Figure 6.21 is subjected to a tensile load; the specimen is 300 mm (12 in.) long, and of square cross section 4.5 mm (0.175 in.) on a side.

(a) Compute the magnitude of the load necessary to produce an elongation of 0.46 mm (0.018 in.).

(b) What will be the deformation after the load is released?

6.29 A cylindrical specimen of aluminum having a diameter of 0.505 in. (12.8 mm) and a gauge length of 2.000 in. (50.800 mm) is pulled in tension. Use the load–elongation characteristics tabulated below to complete problems a through f.

<table>
<thead>
<tr>
<th>Load</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>lbf</td>
<td>in.</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.650</td>
<td>7.330</td>
</tr>
<tr>
<td>3.400</td>
<td>15.100</td>
</tr>
<tr>
<td>5.200</td>
<td>23.100</td>
</tr>
<tr>
<td>6.850</td>
<td>30.400</td>
</tr>
<tr>
<td>7.750</td>
<td>34.400</td>
</tr>
<tr>
<td>8.650</td>
<td>38.400</td>
</tr>
<tr>
<td>9.300</td>
<td>41.300</td>
</tr>
<tr>
<td>10.100</td>
<td>44.800</td>
</tr>
<tr>
<td>10.400</td>
<td>46.200</td>
</tr>
<tr>
<td>10.650</td>
<td>47.300</td>
</tr>
<tr>
<td>10.700</td>
<td>47.500</td>
</tr>
<tr>
<td>10.400</td>
<td>46.100</td>
</tr>
<tr>
<td>10.100</td>
<td>44.800</td>
</tr>
<tr>
<td>9.600</td>
<td>42.600</td>
</tr>
<tr>
<td>8.200</td>
<td>36.400</td>
</tr>
</tbody>
</table>

(a) Plot the data as engineering stress versus engineering strain.

(b) Compute the modulus of elasticity.

(c) Determine the yield strength at a strain offset of 0.002.

(d) Determine the tensile strength of this alloy.

(e) Compute the modulus of resilience.

(f) What is the ductility, in percent elongation?

6.30 A specimen of ductile cast iron having a rectangular cross section of dimensions 4.8 mm × 15.9 mm (⅛ in. × ⅜ in.) is deformed in tension. Using the load–elongation data tabulated below, complete problems a through f.

<table>
<thead>
<tr>
<th>Load</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>lbf</td>
<td>in.</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4.740</td>
<td>1065</td>
</tr>
<tr>
<td>9.140</td>
<td>2055</td>
</tr>
<tr>
<td>12.920</td>
<td>2900</td>
</tr>
<tr>
<td>16.540</td>
<td>3720</td>
</tr>
<tr>
<td>18.300</td>
<td>4110</td>
</tr>
<tr>
<td>20.170</td>
<td>4530</td>
</tr>
<tr>
<td>22.900</td>
<td>5145</td>
</tr>
<tr>
<td>25.070</td>
<td>5635</td>
</tr>
<tr>
<td>26.800</td>
<td>6025</td>
</tr>
<tr>
<td>28.640</td>
<td>6440</td>
</tr>
<tr>
<td>30.240</td>
<td>6800</td>
</tr>
<tr>
<td>31.100</td>
<td>7000</td>
</tr>
<tr>
<td>31.280</td>
<td>7030</td>
</tr>
<tr>
<td>30.820</td>
<td>6930</td>
</tr>
<tr>
<td>29.180</td>
<td>6560</td>
</tr>
<tr>
<td>27.190</td>
<td>6110</td>
</tr>
<tr>
<td>24.140</td>
<td>5430</td>
</tr>
<tr>
<td>18.970</td>
<td>4265</td>
</tr>
</tbody>
</table>

(a) Plot the data as engineering stress versus engineering strain.

(b) Compute the modulus of elasticity.

(c) Determine the yield strength at a strain offset of 0.002.

(d) Determine the tensile strength of this alloy.

(e) Compute the modulus of resilience.

(f) What is the ductility, in percent elongation?

6.31 A cylindrical metal specimen having an original diameter of 12.8 mm (0.505 in.) and gauge length of 50.80 mm (2.000 in.) is pulled in tension until fracture occurs. The diameter at the point of fracture is 6.60 mm (0.260 in.), and the fractured gauge length is 72.14 mm (2.840 in.). Calculate the ductility in terms of
percent reduction in area and percent elongation.

6.32 Calculate the moduli of resilience for the materials having the stress–strain behaviors shown in Figures 6.12 and 6.21.

6.33 Determine the modulus of resilience for each of the following alloys:

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPa</td>
</tr>
<tr>
<td>Steel alloy</td>
<td>550</td>
</tr>
<tr>
<td>Brass alloy</td>
<td>350</td>
</tr>
<tr>
<td>Aluminum alloy</td>
<td>250</td>
</tr>
<tr>
<td>Titanium alloy</td>
<td>800</td>
</tr>
</tbody>
</table>

Use modulus of elasticity values in Table 6.1.

6.34 A brass alloy to be used for a spring application must have a modulus of resilience of at least 0.75 MPa (110 psi). What must be its minimum yield strength?

6.35 (a) Make a schematic plot showing the tensile true stress–strain behavior for a typical metal alloy.

(b) Superimpose on this plot a schematic curve for the compressive true stress–strain behavior for the same alloy. Explain any difference between this curve and the one in part a.

(c) Now superimpose a schematic curve for the compressive engineering stress–strain behavior for this same alloy, and explain any difference between this curve and the one in part b.

6.36 Show that Equations 6.18a and 6.18b are valid when there is no volume change during deformation.

6.37 Demonstrate that Equation 6.16, the expression defining true strain, may also be represented by

\[ \varepsilon_T = \ln \left( \frac{A_0}{A_t} \right) \]

when specimen volume remains constant during deformation. Which of these two expressions is more valid during necking? Why?

6.38 Using the data in Problem 6.29 and Equations 6.15, 6.16, and 6.18a, generate a true stress–true strain plot for aluminum. Equation 6.18a becomes invalid past the point at which necking begins; therefore, measured diameters are given below for the last four data points, which should be used in true stress computations.

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Yield (N)</th>
<th>Length (in.)</th>
<th>Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,400</td>
<td>46,100</td>
<td>2.240</td>
<td>0.461</td>
</tr>
<tr>
<td>10,100</td>
<td>44,800</td>
<td>2.270</td>
<td>0.431</td>
</tr>
<tr>
<td>9,600</td>
<td>42,600</td>
<td>2.300</td>
<td>0.418</td>
</tr>
<tr>
<td>8,200</td>
<td>36,400</td>
<td>2.330</td>
<td>0.370</td>
</tr>
</tbody>
</table>

6.39 A tensile test is performed on a metal specimen, and it is found that a true plastic strain of 0.20 is produced when a true stress of 575 MPa (83,500 psi) is applied; for the same metal, the value of \( K \) in Equation 6.19 is 860 MPa (125,000 psi). Calculate the true strain that results from the application of a true stress of 600 MPa (87,000 psi).

6.40 For some metal alloy, a true stress of 415 MPa (60,175 psi) produces a plastic true strain of 0.475. How much will a specimen of this material elongate when a true stress of 325 MPa (46,125 psi) is applied if the original length is 300 mm (11.8 in.)? Assume a value of 0.25 for the strain-hardening exponent \( n \).

6.41 The following true stresses produce the corresponding true plastic strains for a brass alloy:

<table>
<thead>
<tr>
<th>True Stress (psi)</th>
<th>True Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000</td>
<td>0.10</td>
</tr>
<tr>
<td>60,000</td>
<td>0.20</td>
</tr>
</tbody>
</table>

What true stress is necessary to produce a true plastic strain of 0.25?

6.42 For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strains, prior to necking:

<table>
<thead>
<tr>
<th>Engineering Stress (MPa)</th>
<th>Engineering Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>235</td>
<td>0.194</td>
</tr>
<tr>
<td>250</td>
<td>0.296</td>
</tr>
</tbody>
</table>

On the basis of this information, compute the engineering stress necessary to produce an engineering strain of 0.25.