Chapter 15 - Oscillatory Motion

P15.1  (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the motion is periodic.

(b) To determine the period, we use: \( x = \frac{1}{2}gt^2 \).

The time for the ball to hit the ground is \( t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.904 \text{ s} \).

This equals one-half the period, so \( T = 2(0.904 \text{ s}) = 1.81 \text{ s} \).

(c) The motion is not simple harmonic. The net force acting on the ball is a constant given by \( F = -mg \) (except when it is in contact with the ground), which is not in the form of Hooke's law.

P15.2  (a) \( x = (5.00 \text{ cm}) \cos\left(2t + \frac{\pi}{6}\right) \) \quad \text{At } t = 0, \quad x = (5.00 \text{ cm}) \cos\left(\frac{\pi}{6}\right) = 4.33 \text{ cm}

(b) \( v = \frac{dx}{dt} = -(10.0 \text{ cm/s}) \sin\left(2t + \frac{\pi}{6}\right) \) \quad \text{At } t = 0, \quad v = -5.00 \text{ cm/s}

(c) \( a = \frac{dv}{dt} = -(20.0 \text{ cm/s}^2) \cos\left(2t + \frac{\pi}{6}\right) \) \quad \text{At } t = 0, \quad a = -17.3 \text{ cm/s}^2

(d) \( A = 5.00 \text{ cm} \) and \( T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2}{g}}} = 3.14 \text{ s} \)

P15.3 \( x = (4.00 \text{ m}) \cos(3.00\pi t + \pi) \) Compare this with \( x = A \cos(\omega t + \phi) \) to find

(a) \( \omega = 2\pi f = 3.00\pi \)

or \( f = 1.50 \text{ Hz} \) \quad \text{and} \quad T = \frac{1}{f} = 0.667 \text{ s} \)

(b) \( A = 4.00 \text{ m} \)

(c) \( \phi = \pi \text{ rad} \)

(d) \( x(t = 0.250 \text{ s}) = (4.00 \text{ m}) \cos(1.75\pi) = 2.83 \text{ m} \)
P15.5  
(a) At $t = 0$, $x = 0$ and $v$ is positive (to the right). Therefore, this situation corresponds to $x = A \sin \omega t$

and $\quad v = v_i \cos \omega t$

Since $f = 1.50 \text{ Hz}$, $\omega = 2\pi f = 3.00\pi$

Also, $A = 2.00 \text{ cm}$, so that

\[ x = (2.00 \text{ cm}) \sin 3.00\pi t \]

(b) $v_{\text{max}} = v_i = A \omega = 2.00(3.00\pi) = 6.00\pi \text{ cm/s} = 18.8 \text{ cm/s}$

The particle has this speed at $t = 0$ and next at $t = \frac{T}{2} = \frac{1}{3} \text{ s}$

(c) $a_{\text{max}} = A \omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2 \text{ cm/s}^2 = 178 \text{ cm/s}^2$

This positive value of acceleration first occurs at $t = \frac{3}{4}T = 0.500 \text{ s}$

(d) Since $T = \frac{2}{3} \text{ s}$ and $A = 2.00 \text{ cm}$, the particle will travel 8.00 cm in this time.

Hence, in 1.00 s ($= \frac{3}{2}T$), the particle will travel

\[ 8.00 \text{ cm} + 4.00 \text{ cm} = 12.0 \text{ cm} \]

P15.9  
$x = A \cos \omega t \quad A = 0.05 \text{ m} \quad v = -A \omega \sin \omega t \quad a = -A \omega^2 \cos \omega t$

If $f = 3600 \text{ rev/min} = 60 \text{ Hz}$, then $\omega = 120\pi \text{ s}^{-1}$

$v_{\text{max}} = 0.05(120\pi) \text{ m/s} = 18.8 \text{ m/s}$

$a_{\text{max}} = 0.05(120\pi)^2 \text{ m/s}^2 = 7.11 \text{ km/s}^2$

P15.11  
(a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8.00 \text{ N/m}}{0.500 \text{ kg}}} = 4.00 \text{ s}^{-1}$

so position is given by

\[ x = 10.0 \sin (4.00t) \text{ cm} \]

From this we find that $v = 40.0 \cos (4.00t) \text{ cm/s}$

\[ v_{\text{max}} = 40.0 \text{ cm/s} \]

\[ a = -160 \sin (4.00t) \text{ cm/s}^2 \]

\[ a_{\text{max}} = 160 \text{ cm/s}^2 \]
(b) \[ t = \left( \frac{1}{4.00} \right) \sin^{-1} \left( \frac{x}{10.0} \right) \] and when \( x = 6.00 \text{ cm}, t = 0.161 \text{ s} \).

We find
\[ v = 40.0 \cos \left[ 4.00 \left( 0.161 \right) \right] = 32.0 \text{ cm/s} \]
\[ a = -160 \sin \left[ 4.00 \left( 0.161 \right) \right] = -96.0 \text{ cm/s}^2 \]

(c) Using \( t = \left( \frac{1}{4.00} \right) \sin^{-1} \left( \frac{x}{10.0} \right) \)

when \( x = 0, \ t = 0 \) and when \( x = 8.00 \text{ cm}, t = 0.232 \text{ s} \)

Therefore, \( \Delta t = 0.232 \text{ s} \)

P15.17
(a) \( E = \frac{1}{2} kA^2 = \frac{1}{2} (35.0 \text{ N/m}) (4.00 \times 10^{-2} \text{ m})^2 = 28.0 \text{ mJ} \)

(b) \[ |v| = \sqrt{\frac{k}{m}} A^2 - x^2 = \frac{k}{\sqrt{m}} \sqrt{A^2 - x^2} \]
\[ |v| = \sqrt{\frac{35.0}{50.0 \times 10^{-3}}} \left(4.00 \times 10^{-2}\right)^2 - (1.00 \times 10^{-2})^2 = 1.02 \text{ m/s} \]

(c) \[ \frac{1}{2} m v^2 = \frac{1}{2} k A^2 - \frac{1}{2} k x^2 = \frac{1}{2} (35.0) \left(4.00 \times 10^{-2}\right)^2 - (3.00 \times 10^{-2})^2 \] = 12.2 mJ

(d) \[ \frac{1}{2} k x^2 = E - \frac{1}{2} m v^2 = 15.8 \text{ mJ} \]

P15.18
(a) \[ k = \frac{F}{x} = \frac{20.0 \text{ N}}{0.200 \text{ m}} = 100 \text{ N/m} \]

(b) \[ \omega = \sqrt{\frac{k}{m}} = \sqrt{50.0} \text{ rad/s} \] so \( f = \frac{\omega}{2\pi} = 1.13 \text{ Hz} \)

(c) \[ v_{\text{max}} = \omega A = \sqrt{50.0} (0.200) = 1.41 \text{ m/s} \] at \( x = 0 \)

(d) \[ a_{\text{max}} = \omega^2 A = 50.0 (0.200) = 10.0 \text{ m/s}^2 \] at \( x = \pm A \)

(e) \[ E = \frac{1}{2} k A^2 = \frac{1}{2} (100)(0.200)^2 = 2.00 \text{ J} \]

(f) \[ |v| = \omega \sqrt{A^2 - x^2} = \sqrt{50.0} \sqrt{\frac{8}{9} (0.200)^2} = 1.33 \text{ m/s} \]
(g) \[ |a| = \omega^2 x = 50.0 \left( \frac{0.200}{3} \right) = 3.33 \text{ m/s}^2 \]

P15.20

(a) \[ y_f = y_i + v_i t + \frac{1}{2} a_i t^2 \]
\[ -11 \text{ m} = 0 + 0 + \frac{1}{2} (-9.8 \text{ m/s}^2) t^2 \]
\[ t = \sqrt{\frac{22 \text{ m/s}^2}{9.8 \text{ m}}} = 1.50 \text{ s} \]

(b) Take the initial point where she steps off the bridge and the final point at the bottom of her motion.

\[ (K + U_g + U_s)_i = (K + U_g + U_s)_f \]
\[ 0 + mg y + 0 = 0 + 0 + \frac{1}{2} k x^2 \]
\[ 65 \text{ kg} 9.8 \text{ m/s}^2 36 \text{ m} = \frac{1}{2} k (25 \text{ m})^2 \]
\[ k = \frac{73.4 \text{ N/m}}{} \]

(c) The spring extension at equilibrium is \( x = \frac{F}{k} = \frac{65 \text{ kg} 9.8 \text{ m/s}^2}{73.4 \text{ N/m}} = 8.68 \text{ m} \), so this point is \( 11 + 8.68 \text{ m} = 19.7 \text{ m below the bridge} \) and the amplitude of her oscillation is \( 36 - 19.7 = 16.3 \text{ m} \).

(d) \[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{73.4 \text{ N/m}}{65 \text{ kg}}} = 1.06 \text{ rad/s} \]

(e) Take the phase as zero at maximum downward extension. We find what the phase was 25 m higher, where \( x = -8.68 \text{ m} \):

\[ \text{In } x = A \cos \omega t \]
\[ -8.68 \text{ m} = 16.3 \text{ m} \cos \left( 1.06 \frac{t}{s} \right) \]
\[ 1.06 \frac{t}{s} = -122^\circ = -2.13 \text{ rad} \]
\[ t = -2.01 \text{ s} \]

Then \( +2.01 \text{ s} \) is the time over which the spring stretches.

(f) total time = 1.50 s + 2.01 s = 3.50 s

P15.21 The potential energy is

\[ U_s = \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \cos^2 (\omega t) \]

The rate of change of potential energy is
\[
\frac{dU_s}{dt} = \frac{1}{2} kA^2 \omega \cos(\omega t) \left[ -\omega \sin(\omega t) \right] = -\frac{1}{2} kA^2 \omega \sin 2\omega t
\]

(a) This rate of change is maximal and negative at

\[2\omega t = \frac{\pi}{2}, \quad 2\omega t = 2\pi + \frac{\pi}{2}, \quad \text{or in general,} \quad 2\omega t = 2n\pi + \frac{\pi}{2} \text{ for integer } n\]

Then,

\[t = \frac{\pi}{4\omega} (4n + 1) = \frac{\pi (4n + 1)}{4(3.60 \text{ s}^{-1})}\]

For \(n = 0\), this gives \(t = 0.218 \text{ s}\) while \(n = 1\) gives \(t = 1.09 \text{ s}\).

All other values of \(n\) yield times outside the specified range.

(b) \[
\left| \frac{dU_s}{dt} \right|_{\text{max}} = \frac{1}{2} kA^2 \omega = \frac{1}{2} \left( 3.24 \text{ N/m} \right) \left( 5.00 \times 10^{-2} \text{ m} \right)^2 \left( 3.60 \text{ s}^{-1} \right) = 14.6 \text{ mW}
\]

P15.24

The period in Tokyo is \(T_T = 2\pi \sqrt{\frac{L_T}{g_T}}\)

and the period in Cambridge is \(T_C = 2\pi \sqrt{\frac{L_C}{g_C}}\)

We know \(T_T = T_C = 2.00 \text{ s}\)

For which, we see \(\frac{L_T}{g_T} = \frac{L_C}{g_C}\)

or \(\frac{g_C}{g_T}L_T = \frac{0.994}{0.9927} = 1.0015\)

P15.25

Using the simple harmonic motion model:

\[A = r \theta = 1 \text{ m } 15^\circ = \frac{\pi}{180^\circ} = 0.262 \text{ m}\]

\[\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1 \text{ m}}} = 3.13 \text{ rad/s}\]

(a) \(v_{\text{max}} = A\omega = 0.262 \text{ m } 3.13/\text{s} = 0.820 \text{ m/s}\)

(b) \(a_{\text{max}} = A\omega^2 = 0.262 \text{ m } (3.13/\text{s})^2 = 2.57 \text{ m/s}^2\)
\[ a_{an} = r\alpha \quad \alpha = \frac{a_{an}}{r} = \frac{2.57 \text{ m/s}^2}{1 \text{ m}} = 2.57 \text{ rad/s}^2 \]

FIG. P15.25

(c) \[ F = ma = 0.25 \text{ kg} \times 2.57 \text{ m/s}^2 = 0.641 \text{ N} \]

More precisely,

(a) \[ mgh = \frac{1}{2} mv^2 \quad \text{and} \quad h = L(1 - \cos \theta) \]
\[ \therefore v_{\text{max}} = \sqrt{2gL(1 - \cos \theta)} = 0.817 \text{ m/s} \]

(b) \[ l\alpha = mgL \sin \theta \]
\[ \alpha_{\text{max}} = \frac{mgL \sin \theta}{mL^2} = \frac{g \sin \theta}{L} = 2.54 \text{ rad/s}^2 \]

(c) \[ F_{\text{max}} = mg \sin \theta_i = 0.250(9.80)(\sin 15.0^\circ) = 0.634 \text{ N} \]

The answers agree to two digits. The answers computed from conservation of energy and from Newton’s second law are more precisely correct. With this amplitude the motion of the pendulum is approximately simple harmonic.

P15.28

(a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field of \((9.80 + 5.00) \text{ m/s}^2\). Thus the period is

\[ T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{5.00 \text{ m}}{14.8 \text{ m/s}^2}} = 3.65 \text{ s} \]

(b) \[ T = 2\pi \sqrt{\frac{5.00 \text{ m}}{(9.80 \text{ m/s}^2 - 5.00 \text{ m/s}^2)}} = 6.41 \text{ s} \]

(c) \[ g_{\text{eff}} = \sqrt{(9.80 \text{ m/s}^2)^2 + (5.00 \text{ m/s}^2)^2} = 11.0 \text{ m/s}^2 \]
\[ T = 2\pi \sqrt{\frac{5.00 \text{ m}}{11.0 \text{ m/s}^2}} = 4.24 \text{ s} \]
P15.32  (a) The parallel-axis theorem:

\[ I = I_{cm} + Md^2 = \frac{1}{12}ML^2 + Md^2 = \frac{1}{12}M(1.00\ m)^2 + M(1.00\ m)^2 \]

\[ = M\left(\frac{13}{12}\ m^2\right) \]

\[ T = 2\pi \sqrt{\frac{I}{Mgd}} = 2\pi \sqrt{\frac{M\left(\frac{13}{12}\ m^2\right)}{12M(1.00\ m)}} = 2\pi \sqrt{\frac{13\ m}{12(9.80\ m/s^2)}} = 2.09\ s \]

(b) For the simple pendulum

\[ T = 2\pi \sqrt{\frac{1.00\ m}{9.80\ m/s^2}} = 2.01\ s \]

\[ \text{difference} = \frac{2.09\ s - 2.01\ s}{2.01\ s} = 4.08\% \]

P15.33 \( T = 0.250\ s, \ I = mr^2 = (20.0 \times 10^{-3}\ kg)(5.00 \times 10^{-3}\ m)^2 \)

(a) \[ I = \frac{5.00 \times 10^{-7}\ kg \cdot m^2}{2} \]

(b) \[ \frac{d^2 \theta}{dt^2} = -\kappa \theta; \quad \kappa = \frac{\sqrt{\kappa}}{I} = \omega = \frac{2\pi}{T} \]

\[ \kappa = I \omega^2 = (5.00 \times 10^{-7})(\frac{2\pi}{0.250})^2 = 3.16 \times 10^{-4}\ N \cdot m \cdot rad \]

P15.49 The maximum acceleration of the oscillating system is \( a_{max} = A\omega^2 = 4\pi^2Af^2 \). The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block B is about to slip,

\[ f = f_{max} = \mu_s n = \mu_s mg = m\left(4\pi^2 Af^2\right) \]

\[ A = \frac{\mu_s g}{4\pi^2 f^2} = \frac{0.6(980\ cm/s^2)}{4\pi^2(1.5\ s)} = 6.62\ cm \]

P15.55 We draw a free-body diagram of the pendulum. The force H exerted by the hinge causes no torque about the axis of rotation.

\[ \tau = I\alpha \quad \text{and} \quad \frac{d^2 \theta}{dt^2} = -\alpha \]

\[ \tau = MgL\sin \theta + kx\cos \theta = -I\frac{d^2 \theta}{dt^2} \]

For small amplitude vibrations, use the approximations: \( \sin \theta \approx \theta \), \( \cos \theta \approx 1 \), and \( x \approx s = h\theta \).
Therefore, 
\[ \frac{d^2 \theta}{dt^2} = -\left( \frac{MgL + kh}{l} \right) \theta = -\omega^2 \theta \]

\[ \omega = \sqrt{\frac{MgL + kh}{ML^2}} = 2\pi f \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{MgL + kh}{ML^2}} \]

**P15.57**  (a) At equilibrium, we have

\[ \sum \tau = 0 - mg\left( \frac{L}{2} \right) + kx_0L \]

where \( x_0 \) is the equilibrium compression.

After displacement by a small angle,

\[ \sum \tau = -mg\left( \frac{L}{2} \right) + kxL = -mg\left( \frac{L}{2} \right) + k(x_0 - L\theta)L = -k\theta L^2 \]

But,

\[ \sum \tau = l\alpha = \frac{1}{3} ml^2 \frac{d^2 \theta}{dt^2} \]

So

\[ \frac{d^2 \theta}{dt^2} = -\frac{3k}{m} \theta \]

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with \( \omega^2 = \frac{3k}{m} \).

(b) \[ f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{3k}{m}} = \frac{1}{2\pi} \sqrt{\frac{3(100 \text{ N/m})}{5.00 \text{ kg}}} = 1.23 \text{ Hz} \]

**P15.63**  (a) \[ \sum \vec{F} = -2T \sin \theta \hat{j} \quad \text{where} \quad \theta = \tan^{-1}\left( \frac{y}{L} \right) \]

Therefore, for a small displacement

\[ \sin \theta \approx \tan \theta = \frac{y}{L} \quad \text{and} \quad \sum \vec{F} = \frac{-2Ty}{L} \hat{j} \]
(b) The total force exerted on the ball is opposite in direction and proportional to its displacement from equilibrium, so the ball moves with simple harmonic motion. For a spring system,

$$\sum F = -k\mathbf{x} \quad \text{becomes here} \quad \sum F = -\frac{2T}{L} \dot{y}$$

Therefore, the effective spring constant is $\frac{2T}{L}$ and

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T}{mL}}.$$ 

**P15.67 (a)** When the mass is displaced a distance $x$ from equilibrium, spring 1 is stretched a distance $x_1$ and spring 2 is stretched a distance $x_2$.

By Newton’s third law, we expect

$$k_1x_1 = k_2x_2.$$ 

When this is combined with the requirement that

$$x = x_1 + x_2,$$

we find

$$x_1 = \left[ \frac{k_2}{k_1 + k_2} \right] x.$$ 

The force on either spring is given by

$$F_i = \left[ \frac{k_2}{k_1 + k_2} \right] x = ma$$

where $a$ is the acceleration of the mass $m$.

This is in the form

$$F = k_{eff}x = ma$$

and

$$T = 2\pi \sqrt{\frac{m}{k_{eff}}} = 2\pi \sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}.$$ 

(b) In this case each spring is distorted by the distance $x$ which the mass is displaced. Therefore, the restoring force is

$$F = -(k_1 + k_2)x \quad \text{and} \quad k_{eff} = k_1 + k_2$$

so that

$$T = 2\pi \sqrt{\frac{m}{(k_1 + k_2)}}.$$