Chapter 19 - Temperature

P19.1 Since we have a linear graph, the pressure is related to the temperature as \( P = A + BT \), where \( A \) and \( B \) are constants. To find \( A \) and \( B \), we use the data

\[
\begin{align*}
0.900 \text{ atm} &= A + (-80^\circ \text{C})B \\
1.635 \text{ atm} &= A + (78^\circ \text{C})B
\end{align*}
\]

Solving (1) and (2) simultaneously, \( 1.635 - 0.900 = 78B + 80B \) we find

\[
B = 4.652 \times 10^{-3} \text{ atm/}^\circ \text{C}
\]

and \( A = 1.272 \text{ atm} \)

Therefore, \( P = 1.272 \text{ atm} + (4.652 \times 10^{-3} \text{ atm/}^\circ \text{C})T \)

(a) At absolute zero \( P = 0 = 1.272 \text{ atm} + (4.652 \times 10^{-3} \text{ atm/}^\circ \text{C})T \)

which gives \( T = -274^\circ \text{C} \)

(b) At the freezing point of water \( P = 1.272 \text{ atm} + 0 = 1.27 \text{ atm} \).

(c) And at the boiling point \( P = 1.272 \text{ atm} + (4.652 \times 10^{-3} \text{ atm/}^\circ \text{C})(100^\circ \text{C}) = 1.74 \text{ atm} \).

P19.3 (a) \( T_c = \frac{9}{5}T_c + 32.0^\circ F = \frac{9}{5}(-195.81) + 32.0 = -320^\circ F \)

(b) \( T = T_c + 273.15 = -195.81 + 273.15 = 77.3 \text{ K} \)

P19.5 The wire is 35.0 m long when \( T_c = -20.0^\circ \text{C} \).

\[
\Delta L = L_\alpha (T - T_i)
\]

\( \alpha = \alpha (20.0^\circ \text{C}) = 1.70 \times 10^{-5} \text{ (C}^\circ)^{-1} \text{ for Cu.} \)

\[
\Delta L = (35.0 \text{ m})(1.70 \times 10^{-5} \text{ (C}^\circ)^{-1})(35.0^\circ \text{C} - (-20.0^\circ \text{C})) = 3.27 \text{ cm}
\]

P19.6 Each section can expand into the joint space to the north of it. We need think of only one section expanding. \( \Delta L = L_\alpha \Delta T = (25.0 \text{ m})(12.0 \times 10^{-6}/\text{C}^\circ)(40.0^\circ \text{C}) = 1.20 \text{ cm} \)
\[ L_{\text{Al}} (1 + \alpha_{\text{Al}} \Delta T) = L_{\text{Brass}} (1 + \alpha_{\text{Brass}} \Delta T) \]

\[ \Delta T = \frac{L_{\text{Al}} - L_{\text{Brass}}}{L_{\text{Brass}} \alpha_{\text{Brass}} - L_{\text{Al}} \alpha_{\text{Al}}} = \frac{(10.01 - 10.00)}{(10.00)(19.0 \times 10^{-6}) - (10.01)(24.0 \times 10^{-6})} \]

\[ \Delta T = -199^\circ C \text{ so } T = -179^\circ C \]

This is attainable, because it is above absolute zero.

(b) \[ \Delta T = \frac{(10.02 - 10.00)}{(10.00)(19.0 \times 10^{-6}) - (10.02)(24.0 \times 10^{-6})} \]

\[ \Delta T = -396^\circ C \text{ so } T = -376^\circ C, \text{ which is below } 0 \text{ K so it cannot be reached.} \]

The rod and ring cannot be separated by changing their temperatures together.

\[ \Delta V = V_i \beta_i \Delta T - V_{\text{Al}} \beta_{\text{Al}} \Delta T = (\beta_i - 3 \alpha_{\text{Al}}) V_i \Delta T \]

\[ = (9.00 \times 10^{-4} - 0.720 \times 10^{-4}) \text{C}^{-1} \left(2 \text{,}000 \text{ cm}^3\right)(60.0^\circ C) \]

\[ \Delta V = 99.4 \text{ cm}^3 \text{ overflows.} \]

(b) The whole new volume of turpentine is

\[ 2 \, 000 \text{ cm}^3 + 9.00 \times 10^{-4} \text{C}^{-1} \left(2 \text{,}000 \text{ cm}^3\right)(60.0^\circ C) = 2 \, 108 \, \text{cm}^3 \]

so the fraction lost is \[ \frac{99.4 \text{ cm}^3}{2 \, 108 \text{ cm}^3} = 4.71 \times 10^{-2} \]

and this fraction of the cylinder’s depth will be empty upon cooling:

\[ 4.71 \times 10^{-2} (20.0 \text{ cm}) = 0.943 \text{ cm} \]

\[ (a) \text{ Initially, } P_i V_i = n_i RT_i \]

\[ (1.00 \text{ atm}) V_i = n_i R (10.0 + 273.15) \text{ K} \]

\[ \text{Finally, } P_f V_f = n_f RT_f \]

\[ P_f (0.280V_f) = n_f R (40.0 + 273.15) \text{ K} \]

Dividing these equations,

\[ \frac{0.280P_f}{1.00 \text{ atm}} = \frac{313.15 \text{ K}}{283.15 \text{ K}} \]

\[ P_f = 3.95 \text{ atm} \]

or

\[ P_f = 4.00 \times 10^5 \text{ Pa(abs.)} \]

(b) After being driven

\[ P_d (1.02)(0.280V_f) = n_i R (85.0 + 273.15) \text{ K} \]

\[ P_d = 1.121P_f = 4.49 \times 10^5 \text{ Pa} \]
P19.19 The equation of state of an ideal gas is $PV = nRT$ so we need to solve for the number of moles to find $N$.

$$n = \frac{PV}{RT} = \frac{(1.01 \times 10^5 \text{ N/m}^2)(10.0 \text{ m})(20.0 \text{ m})(30.0 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 2.49 \times 10^5 \text{ mol}$$

$$N = nN_A = 2.49 \times 10^5 \text{ mol} (6.022 \times 10^{23} \text{ molecules/mol}) = 1.50 \times 10^{29} \text{ molecules}$$

P19.21

$$\sum F_y = 0: \quad \rho_{\text{out}} g V - \rho_{\text{in}} g V - (200 \text{ kg}) g = 0$$

$$\left(\rho_{\text{out}} - \rho_{\text{in}}\right) (400 \text{ m}^3) = 200 \text{ kg}$$

The density of the air outside is $1.25 \text{ kg/m}^3$.

From $PV = nRT$, $\frac{n}{V} = \frac{P}{RT}$. This equation means that at constant pressure the density is inversely proportional to the temperature. Then the density of the hot air is

$$\rho_{\text{in}} = (1.25 \text{ kg/m}^3) \left(\frac{283 \text{ K}}{T_{\text{in}}}\right)$$

Then

$$\left(1.25 \text{ kg/m}^3\right) \left(1 - \frac{283 \text{ K}}{T_{\text{in}}}\right) (400 \text{ m}^3) = 200 \text{ kg}$$

$$1 - \frac{283 \text{ K}}{T_{\text{in}}} = 0.400$$

$$0.600 = \frac{283 \text{ K}}{T_{\text{in}}} \quad T_{\text{in}} = 472 \text{ K}$$

P19.33 Neglecting the expansion of the glass,

$$\Delta h = \frac{V}{A} \beta \Delta T$$

$$\Delta h = \frac{\frac{5}{3} \pi \left(2.50 \text{ cm}^3\right)}{\pi \left(2.00 \times 10^{-3} \text{ cm}^3\right)} \left(1.82 \times 10^{-4} \text{ C}^{-1}\right) = 3.55 \text{ cm}$$

P19.34 (a) The volume of the liquid increases as $\Delta V_L = V_L \beta \Delta T$. The volume of the flask increases as $\Delta V_g = 3 \alpha V L \Delta T$. Therefore, the overflow in the capillary is $V_c = V_L \Delta T (\beta - 3 \alpha)$; and in the capillary $V_c = A \Delta h$.

Therefore, $\Delta h = \frac{V_L}{A} (\beta - 3 \alpha) \Delta T$

(b) For a mercury thermometer $\beta (\text{Hg}) = 1.82 \times 10^{-4} \text{ C}^{-1}$
and for glass, $3\alpha = 3 \times 3.20 \times 10^{-6} \circ C^{-1}$

Thus $\beta - 3\alpha \approx \beta$ within better than 6%. The value of $\alpha$ is typically so small compared to $\beta$ that it can be ignored in the equation for a good approximation.

P19.38 (a) $\frac{P_0 V}{T} = \frac{P' V'}{T'}$

$V' = V + Ah$

$P' = P_0 + \frac{kh}{A}$

$\left( P_0 + \frac{kh}{A} \right) (V + Ah) = P_0 V \left( \frac{T'}{T} \right)$

$\left( 1.013 \times 10^5 \text{ N/m}^2 + 2.00 \times 10^5 \text{ N/m}^3 h \right)$

$\left( 5.00 \times 10^{-3} \text{ m}^3 + (0.010 \text{ m}^3) h \right)$

$= (1.013 \times 10^5 \text{ N/m}^2) (5.00 \times 10^{-3} \text{ m}^3) \left( \frac{523 \text{ K}}{293 \text{ K}} \right)$

$2000h^2 + 2013h - 397 = 0$

$h = \frac{-2013 \pm 2689}{4000} = 0.169 \text{ m}$

(b) $P' = P + \frac{kh}{A} = 1.013 \times 10^5 \text{ Pa} + \frac{(2.00 \times 10^5 \text{ N/m}) (0.169)}{0.010 \text{ m}^2}$

$P' = 1.35 \times 10^5 \text{ Pa}$

P19.39 (a) We assume that air at atmospheric pressure is above the piston.

In equilibrium $P_{gas} = \frac{mg}{A} + P_0$

Therefore, $\frac{nRT}{hA} = \frac{mg}{A} + P_0$

or $h = \frac{nRT}{mg + P_0 A}$

where we have used $V = hA$ as the volume of the gas.

(b) From the data given,

$h = \frac{0.200 \text{ mol} (8.314 \text{ J/K} \cdot \text{mol}) (400 \text{ K})}{20.0 \text{ kg} (9.80 \text{ m/s}^2) + (1.013 \times 10^5 \text{ N/m}^2) (0.008 \text{ m}^2)}$

$= 0.661 \text{ m}$
**P19.40** The angle of bending $\theta$, between tangents to the two ends of the strip, is equal to the angle the strip subtends at its center of curvature. (The angles are equal because their sides are perpendicular, right side to the right side and left side to left side.)

(a) The definition of radian measure gives

\[ L_i + \Delta L_i = \theta r_i \]

and

\[ L_i + \Delta L_2 = \theta r_2 \]

By subtraction,

\[ \Delta L_2 - \Delta L_i = \theta (r_2 - r_i) \]

\[ \alpha_2 L_i \Delta T - \alpha_1 L_i \Delta T = \theta \Delta r \]

\[ \theta = \frac{(\alpha_2 - \alpha_1) L_i \Delta T}{\Delta r} \]

(b) In the expression from part (a), $\theta$ is directly proportional to $\Delta T$ and also to $(\alpha_2 - \alpha_1)$. Therefore $\theta$ is zero when either of these quantities becomes zero.

(c) The material that expands more when heated contracts more when cooled, so the bimetallic strip bends the other way. It is fun to demonstrate this with liquid nitrogen.

(d) \[
\theta = \frac{2(\alpha_2 - \alpha_1) L_i \Delta T}{2\Delta r} = \frac{2\left(19 \times 10^{-6} - 0.9 \times 10^{-6}\right)^\circ C^{-1}}{(200 \text{ mm})(1^\circ C)} \times \frac{0.500 \text{ mm}}{180^\circ} = 0.830^\circ \]

**P19.41** From the diagram we see that the change in area is

\[ \Delta A = \Delta w + w \Delta \ell + \Delta w \Delta \ell \]

Since $\Delta \ell$ and $\Delta w$ are each small quantities, the product $\Delta w \Delta \ell$ will be very small. Therefore, we assume $\Delta w \Delta \ell \approx 0$.

Since $\Delta w = \omega \Delta T$ and $\Delta \ell = \ell \alpha \Delta T$,

we then have

\[ \Delta A = \ell w \alpha \Delta T + \ell \omega \Delta T \]

and since $A = \ell w$, \[ \Delta A = 2 \alpha A \Delta T \]

The approximation assumes $\Delta w \Delta \ell \approx 0$, or $\alpha T \approx 0$. Another way of stating this is $\alpha T \ll 1$. 
**P19.44**  
(a) \[ T = 2\pi \frac{L}{g} \]  
so \[ L_i = \frac{T_i^2 g}{4\pi^2} = \frac{(1.00 \text{ s})^2 (9.80 \text{ m/s}^2)}{4\pi^2} = 0.248 \text{ m} \]

\[ \Delta L = \alpha L \Delta T = 19.0 \times 10^{-6} \text{C}^{-1} (0.248 \text{ m})(10.0 \text{C}) = 4.72 \times 10^{-5} \text{ m} \]

\[ T_f = 2\pi \sqrt{\frac{L_i + \Delta L}{g}} = 2\pi \sqrt{\frac{0.248 \text{ m}}{9.80 \text{ m/s}^2}} = 1.00 \text{ s} \]

\[ \Delta T = 9.50 \times 10^{-5} \text{ s} \]

(b) In one week, the time lost \[ = 1 \text{ week } (9.50 \times 10^{-5} \text{ s lost per second}) \]

\[ = (7.00 \text{ d/week}) \left( \frac{86400 \text{ s}}{1.00 \text{ d}} \right) (9.50 \times 10^{-5} \text{ s lost}) \]

\[ \text{time lost} = 57.5 \text{ s lost} \]

**P19.47** After expansion, the length of one of the spans is

\[ L_f = L_i (1 + \alpha \Delta T) = 125 \text{ m} \left[ 1 + 12 \times 10^{-6} \text{C}^{-1} (20.0 \text{C}) \right] = 125.03 \text{ m} \]

\[ L_f, y, \text{ and the original 125 m length of this span form a right triangle with } y \text{ as the altitude. Using the Pythagorean theorem gives:} \]

\[ (125.03 \text{ m})^2 = y^2 + (125 \text{ m})^2 \]

yielding \[ y = 2.74 \text{ m} \].

**P19.52**  
(a) No torque acts on the disk so its angular momentum is constant. Its moment of inertia decreases as it contracts so its angular speed must increase.

(b) \[ I_i \omega_i = I_f \omega_f = \frac{1}{2} MR_i^2 \omega_i = \frac{1}{2} MR_f^2 \omega_f = \frac{1}{2} M (R_i + R_f \alpha \Delta T)^2 \omega_f = \frac{1}{2} MR_f^2 [1 - \alpha \Delta T]^2 \omega_f \]

\[ \omega_f = \omega_i [1 - \alpha \Delta T]^2 = \frac{25.0 \text{ rad/s}}{(1 - (17 \times 10^{-6} / \text{C}^o)(830\text{C}))} \approx 25.0 \text{ rad/s} \]

\[ \approx 25.7 \text{ rad/s} \]

**P19.54** With piston alone: \[ T = \text{constant, so } PV = P_0 V_0 \]

or \[ P(A h_1) = P_0 (A h_0) \]

With \[ A = \text{constant,} \]

\[ P = P_0 \left( \frac{h_0}{h} \right) \]

But,

\[ P = P_0 + \frac{m_p g}{A} \]

where \[ m_p \] is the mass of the piston.
Thus,

\[ P_0 + \frac{m_p g}{A} = P_i \left( \frac{h_0}{h_i} \right) \]

which reduces to

\[ h_i = \frac{h_0}{1 + \frac{m_p g}{P_i A}} = \left( \frac{50.0 \text{ cm}}{1 + 20.0 \text{ kg} \left( 9.80 \text{ m/s}^2 \right) \left[ 1.013 \times 10^5 \text{ Pa} \pi \left( 0.400 \text{ m} \right)^2 \right]} \right) = 49.81 \text{ cm} \]

With the dog of mass \( M \) on the piston, a very similar calculation (replacing \( m_p \) by \( m_p + M \)) gives:

\[ h' = \frac{h_0}{1 + \frac{(m_p + M) g}{P_i A}} = \left( \frac{50.0 \text{ cm}}{1 + 95.0 \text{ kg} \left( 9.80 \text{ m/s}^2 \right) \left[ 1.013 \times 10^5 \text{ Pa} \pi \left( 0.400 \text{ m} \right)^2 \right]} \right) = 49.10 \text{ cm} \]

Thus, when the dog steps on the piston, it moves downward by

\[ \Delta h = h_i - h' = 49.81 \text{ cm} - 49.10 \text{ cm} = 0.706 \text{ cm} = 7.06 \text{ mm} \]

(b) \( P = \text{const} \), so

\[ \frac{V}{T} = \frac{V'}{T'} \quad \text{or} \quad \frac{Ah_i}{T} = \frac{Ah'}{T'} \]

giving

\[ T = T_i \left( \frac{h_i}{h'} \right) = 293 \text{ K} \left( \frac{49.81}{49.10} \right) = 297 \text{ K} \]

(or 24°C)