Chapter 02
Motion, Momentum and Energy

Newton's Third Law of Motion

Newton's Third Law of Motion states that for each force (or for the net force) $F_{AB}$ that a body A exerts on a body B, the body B exerts a force (a counterforce) $F_{BA} = -F_{AB}$ on body A. Notice the two forces (the original force and the counterforce) have the same magnitude but are pointing in opposite directions. Multiplying a vector by -1 turns the vector to the opposite direction without changing the magnitude.

$$F_{AB} = -F_{BA}$$

The combination of these two laws—Newton's Second and Third Laws of Motion—is very powerful in that they can be used as fundamental physical laws (postulates) to logically derive many derived physical laws (theorems) of science. (Newton's First Law of Motion can actually be derived as a theorem from them.)

Linear Momentum

Linear momentum is the product of mass and velocity $mv$. This quantity is often denoted by $p$ in physics textbooks. $p=mv$. Since mass is a scalar and the velocity $v$ is a vector, the product (of this scalar and vector) is a vector pointing in the same direction as the velocity. This simple definition of linear momentum had subtle complications in later theories of physics. For example, when Einstein developed his “theory of special relativity,” there was a serious discussion regarding what kind of mass is used in the equation $p=mv$ and even variations of the $p=mv$ equation were suggested. Such subtle concepts are best avoided until special relativity is discussed.
Linear Momentum and Newton's Laws

Chapter 01 introduced Newton's Second Law of Motion as $F_{\text{net}}=ma$ but Newton actually stated his Second Law of Motion in a more general form relating the net force to the rate of change $\Delta p / \Delta t$ of the linear momentum.

$$F_{\text{net}} = K \frac{\Delta p}{\Delta t}$$

where $K$ is a constant value that depends on the units chosen for the quantities. Remember that the capital Greek letter delta “$\Delta$” indicates a change. Thus,

$$\Delta p = \text{the final linear momentum} – \text{the initial linear momentum} = \text{change in } p$$

and

$$\Delta t = \text{the final time} – \text{the initial time} = \text{change in time}.$$  

Scientists and engineers prefer to use units that make the constant $K = 1$ so it disappears from the equation. Actually, there is an argument which suggests we can consider the constant $K = 1$ no matter what units are used.

$$F_{\text{net}} = \frac{\Delta p}{\Delta t} \text{ (for K=1)}$$

An important point is that if the net force is zero (the null vector), then Newton’s Second Law of Motion shows us that the linear momentum remains constant rather than changing.

Impulse

Stopping or slowing an object with linear momentum requires that a force acts on the object for a sufficient time. Speeding the object to a certain speed also requires a force acting for sufficient time. “Impulse” $I_m$ is the product of net force and the time over which that force acts.

$$I_m = F_{\text{net}} \Delta t$$

We can use a little algebra to rewrite Newton's Second Law of Motion as

$$I_m = K \Delta p = F_{\text{net}} \Delta t$$

or, since we like to choose units such that $K = 1$,

$$I_m = \Delta p = F_{\text{net}} \Delta t \text{ (for K=1)}$$

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Thus, we see that the impulse causes a change in linear momentum and impulse is equal to the change in linear momentum. Without impulses, the linear momentum does not change. Without impulse, the linear momentum of an object stays constant.

**Systems of Objects**

Thus far, the discussion has focused on single objects. However, we can generalize our concepts to include collections of objects. A physical system is a collection of physical objects. Such a system has a total linear momentum \( P_{\text{total}} \) which is the vector sum of all the linear momentums of the objects in the system. The system has a total force \( F_{\text{total}} \) which is the vector sum of all the forces acting on the objects in the system. In terms of these totals, Newton's Second Law of Motion for the entire system can be written as

\[
F_{\text{total}} = K \frac{\Delta P_{\text{total}}}{\Delta t}
\]

\[
F_{\text{total}} = \frac{\Delta P_{\text{total}}}{\Delta t} \quad \text{(for } K=1)\]

The impulse equation for the entire system of objects becomes

\[
I_m = F_{\text{total}} \Delta t = K \Delta P_{\text{total}}
\]

\[
I_m = F_{\text{total}} \Delta t = \Delta P_{\text{total}} \quad \text{(for } K=1)\]

These equations for systems of objects look like the equations we had for a single object. Thus, some conclusions can be made, for systems, that are similar to conclusions for a single object. For example, if the total force (the vector sum) on a system is zero (the null vector) then the total linear momentum of the system remains constant. If the impulse on the system is zero, then its total linear momentum remains constant. When the total linear momentum remains constant, we say the linear momentum is “conserved.”

In order to describe more physics of a whole system of objects, we should introduce some classification of systems. A “closed” system is a physical system for which the collection of objects does not change. No matter flows into or out of a closed system. The amount of matter stays constant. We consider only objects internal to the system. The objects in a closed system can interact. They can push or pull on each other, and they can undergo chemical reactions with each other, but they don't leave the closed system.
Objects in a system may influence each other by exerting forces on each other or undergoing chemical reactions with each other. An “isolated” system is one on which only objects in the system exert forces on the objects in the system. No external objects exert forces on parts of the isolated system. Some textbooks can be very strict on this matter by saying the system is isolated if there are no external influences on the system including forces, external “fields” and communications with any part of the system.

Consider any closed system. The parts of the system (the objects in the system) can interact with each other by exerting forces on each other. However, Newton's Third Law of Motion assures us that if one part exerts a force on a second part, the second part exerts a force with the same magnitude on the first but in the opposite direction. Thus, the vector sum of these forces will be zero. Thus, the interactions between parts of the closed system cannot change the total linear momentum of the system. Only a net force $F_{\text{external}}$ from sources outside the closed system can change the total linear momentum of the system. Thus, Newton's Second Law for a closed system can be written

$$F_{\text{external}} = K \Delta P_{\text{total}} / \Delta t \quad \text{(closed system)}$$

$$F_{\text{external}} = \Delta P_{\text{total}} / \Delta t \quad \text{for K=1 and closed system)}$$

Next, suppose the closed system is either also an isolated system or at least the net external force on the system is zero. Without net external force, the total linear momentum of the system is conserved. This can be stated as a physical law called the “Law of Conservation of Linear Momentum.”

**Law of Conservation of Linear Momentum**: If the net external force $F_{\text{external}}$ on a closed system is zero, then the total linear momentum of the system is conserved.

Thus, we have seen a derived law logically following from Newton's Laws of Motion. The Law of Conservation of Linear Momentum can be viewed as a theorem that follows from the fundamental or basic physical laws which are Newton's Laws of Motion. With zero net external force, a closed system behaves much like a closed and isolated system. You should realize that I never stated how many objects you can have in a system. A system can have any number of objects or parts. Thus, one object can be the only one in a system, but you can also have 20 million objects in another system. Thus, the Law of Conservation of Linear Momentum applies to systems and also applies to single objects. If the net force on a body is zero, the linear momentum of the body is conserved.
Two Bodies Pushing on Each Other

Consider the collision of two bodies without any net forces other than those the bodies exert on each other. The total linear momentum of the two bodies must be conserved. Thus, the total linear momentum after collision is the same as it was before collision. Consider the example of a body with mass 10 kg moving at 20 m/s to the right until it strikes and sticks to a second body at rest with mass 30 kg. If the only nonzero net forces in the problem are those caused by the collision, then the total linear momentum must be conserved. The total linear momentum before the collision was

\[ 10 \text{ kg} \times 20 \text{ m/s to the right} = 200 \text{ kg m/s to the right} \]

Thus, the total linear momentum after the collision must be the same 200 kg m/s to the right. The two bodies are stuck together after the collision so they must move together. If \( v \) is the velocity of the pair after collision, then the total linear momentum is

\[ (10 \text{ kg} + 30 \text{ kg}) \cdot v = 40 \text{ kg} \cdot v \]

Thus,

\[ 40 \text{ kg} \cdot v = 200 \text{ kg m/s to the right} \]

\[ v = 50 \text{ m/s to the right} \]

If two bodies at first are at rest relative to each other but then push on each other, the conservation of linear momentum implies the two bodies will have linear momentums with the same magnitude but opposite directions. Thus, if the two bodies have masses 10 kg and 30 kg are stuck to each other and at rest until one mass pushes the other away so the 10-kg mass is traveling 50 m/s to the left, then the 10-kg mass has linear momentum

\[ 10 \text{ kg} \times 50 \text{ m/s to the left} = 500 \text{ kg m/s to the left}. \]

Since the linear momentum must be conserved, the 40-kg body must have 500 kg m/s to the right as its linear momentum. Thus, the 40-kg mass must be moving with a velocity of

\[ \frac{500 \text{ kg m/s to the right}}{40 \text{ kg}} = 12.5 \text{ m/s to the right}. \]

Projections or Shadows of Vectors

Much of the physics we have introduced thus far has involved vectors. Some arithmetic with vectors involves a branch of mathematics called “trigonometry.” In this course, we avoid involvement of trigonometry, but sometimes a few facts of trigonometry aid discussions involving vectors. Of special
interest to some calculations in physics is the “projection” or “shadow” \( \mathbf{V}_x \) of a vector \( \mathbf{V} \) along a specified direction \( \mathbf{x} \). Projections of vectors along \( \mathbf{x} \) are shown in the following three diagrams.

Some textbooks regard the vector \( \mathbf{V}_x \) as being the projection or shadow of vector \( \mathbf{V} \). However, some books treat the projection as a scalar \( \mathbf{V}_x \) with the same units as \( \mathbf{V} \) has and equal to either the magnitude of \( \mathbf{V}_x \) or the product of -1 and this magnitude. For example, in the first diagram, \( \mathbf{V}_x \) is positive for the vector \( \mathbf{V} \) as shown because the vector \( \mathbf{V}_x \) is along the \( \mathbf{x} \) direction. For the middle diagram, \( \mathbf{V}_x \) is
negative because the projection or shadow is in a direction opposite the \( x \) direction. For the last diagram, \( V_x \) is zero because the projection or shadow along the \( x \) direction has zero magnitude. Notice that the projection decreases in magnitude as the angle approaches 90 degrees.

There is a function in trigonometry, called the “cosine” of an angle, that allows us to calculate the value of \( V_x \) for a given vector \( \mathbf{V} \) and direction \( x \). We don't need to know all the properties of this cosine function in this course, but there are a few properties that may help us understand the projection of a vector along a direction. Notice the angle \( \theta \) from the direction \( x \) to the vector \( \mathbf{V} \) as marked in the diagrams. The cosine of this angle is often abbreviated as “\( \cos \theta \)” in writing equations. A formula you may find useful is

\[
V_x = V \cos \theta
\]

where \( V \) is the magnitude of vector \( \mathbf{V} \) and the angle \( \theta \) is from direction \( x \) to vector \( \mathbf{V} \). The cosine of zero degrees is one and the cosine of 90 degrees is 0. For angles larger than 90 degrees but smaller than 180 degrees, the cosine is negative.

\[
\begin{align*}
\cos 0 &= 1 \\
\cos 90^\circ &= 0 \\
\cos 180^\circ &= -1
\end{align*}
\]

Thus, the cosine decreases as the angle opens from zero toward 90 degrees. This cosine function contains the behavior of the projection (including the sign of the projection when the projection is viewed as a scalar quantity \( V_x \) instead of a vector \( \mathbf{V}_x \)).

Your instructor suggested that you should buy a scientific calculator for about $13 at a store or install spreadsheet software on your computer. A typical scientific calculator calculates cosines of angles. (By default, typical calculators assume the angles you enter are in degrees.) Your instructor can show you how to use most popular calculators to calculate cosines. You can also calculate cosines using spreadsheet software on your computer, but you must be warned that typical spreadsheet software assumes you are entering angles in “radians” rather than “degrees.” Your instructor can show you how to convert from degrees to radians.
Work on a Body

Work W done on a body is commonly described as the product of the net or total force \( \mathbf{F} \) on the body and the distance the body moves while that net force is applied. Actually the work \( W \) is defined as the product of the magnitude \( F \) of the force, the magnitude \( x \) of the displacement and the cosine of the angle \( \theta \) between the force and displacement vectors.

\[
W = F \times x \times \cos \theta
\]

We can say work is the “dot product” or “scalar product” of the displacement \( \mathbf{x} \) and the force \( \mathbf{F} \) since the dot or scalar product of two vectors is defined as the product of the magnitudes and the cosine of the angle between them. We can also say the work is the product of the displacement and the projection \( F_x \) of the force \( \mathbf{F} \) along the direction of the displacement \( \mathbf{x} \).

The work is a scalar. However, force and displacement are vectors and the work done depends on the directions of the net force applied and the displacement of the body. This can also be said to be the product of the magnitude \( x \) of the displacement and the scalar projection or shadow \( F_x \) of the net force \( \mathbf{F} \) in the direction of the displacement vector. This is shown in the following three figures for a net force \( \mathbf{F} \) and a displacement \( \mathbf{x} \). Each figure shows a different case, including

- with the (vector) projection \( F_x \) in the same direction as the displacement,
- the (vector) projection \( F_x \) in the direction opposite the displacement and
- the (vector) projection \( F_x \) being zero.
Since force and displacement have different units, the lengths of the force vectors in the diagrams should not be compared with the length of the displacement vector. When the projection $F_x$ and the displacement $x$ vectors are in the same direction, the work $W$ done on the body is positive. When the projection and the displacement are in opposite directions, the work done on the body is negative. We can view negative work on the body as positive work done by the body. The last of the three figures shows the net force $F$ perpendicular to the displacement. This results in zero projection $F_x$ and, thus, zero work done on the body.
The projection allows us to consider the situation as a one-dimensional situation. With only one dimension, there are only two directions: to the right which we call the positive direction, and to the left which we call the negative direction. Thus, positive five meters is five meters to the right and -5 meters is five meters to the left. With such a system, we can write the work $W$ as the product

$$W = F_x x$$

with the signs of $F_x$ and $x$ giving the directions (right or left). If the net force or the projection of the net force is positive (which means it is to the right) and the displacement is also positive, then the work (which is the product $F_x x$) is also positive. If either one of $F_x$ or $x$ is positive and the other is negative (indicating they are in opposite directions), then the work is negative. If $F_x$ and $x$ are both negative, then the work (which is the product of $F_x$ and $x$) is positive.

Mechanical devices that perform work are often called “machines.” Mechanical engineers design devices that do significant work with little force. Since work is a product of force and displacement, for an amount of work done the displacement must be larger for the force to be smaller. Devices that accomplish this include levers with fulcrums, and pulleys. A “lever” is a long object (usually a rod, beam or pipe) that has one fixed part mounted on a rigid object called a “fulcrum” so that the lever can be rotated on or about the fulcrum while the mounted part of the lever remains at the fulcrum. In typical use, the lever has a “short” end that protrudes a small distance past the fulcrum and a “long” end protruding a longer distance from the fulcrum. The short end is used to move a heavy or stuck object a small distance by moving the end of the long end a greater distance. The force required to move long end of the lever a larger distance is much smaller than the force applied by the short end to the object. The same amount of work is done at either end of the lever, but the force applied at one end was smaller because the displacement was larger.

Systems of pulleys use a similar concept. A rope pulled through the pulleys can apply a large force to cause a heavy or stuck object to move a small distance, but does so by applying a smaller force to pull rope through the pulleys. The length of the travel of the object is smaller than the length of rope that must be pulled through the pulley system to achieve the large force that moves the object. Thus, the same amount of work is done on the rope as is done in moving the object.

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Work on a Closed System and Energy

The work on a single body could be positive, negative or zero. A closed system is composed of bodies which we call components or parts of the system. The total work $W_{\text{total}}$ done on a closed system is the sum of all the work done by external forces on the components of the system. This total work done can be positive, negative or zero. Suppose we can number all the parts of a system. Let $W_n$ be the work done on part $n$ of the system. ($n = 1, 2, 3, \ldots$) Then the total work done on the system will be

$$W_{\text{total}} = W_1 + W_2 + W_3 + \ldots$$

The concept of work is closely related to the concept of energy. Energy of a system is part of the description of the state of the system. Energy is often described as the ability of a body or system to do work. The energy is a relative quantity because the energy of a state of the system must be specified with respect to some reference state. To make the description of energy more precise, I usually adopt the following definition of energy.

The energy of a system in its current state, relative to the energy of a reference state, is the minimum total work that must be done on the system (by external forces) to bring it from the reference state to the current state.

Consider a spaceship at rest with an empty fuel tank so the spaceship needs to be pushed back to a space station to refuel. Suppose a rocket pushes the spaceship along a straight line with a force $F$ with magnitude $F$ through a displacement $x$ with magnitude $x$ so that the rocket does work $W_{\text{spaceship}}$ on the spaceship. The work done on the spaceship is

$$W_{\text{spaceship}} = F \times$$

and we have increased the energy of the spaceship by the amount of work $W_{\text{spaceship}}$ done on it. The energy of the moving spaceship relative to the state of being at rest is equal to this amount of work done to bring the spaceship from rest to moving along a line. The energy gained is called “kinetic energy” because it is energy due to the motion. More about this type of energy is explained later in this Lecture Summary.

Work and energy are are measured in joules. A joule is a kg m$^2$/s$^2$. It is also a newton meter. A joule is commonly represented by the capital letter $J$.

$$J = \text{kg m}^2/\text{s}^2 = \text{N m}$$

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From the definition of energy I chose, the work-energy theorem follows.

**Work-energy theorem:** The change in energy of a closed system is equal to the total work done (by external forces) on the system.

Since this is a theorem, it is a derived physical law. Notice that this total work can be negative. Negative work on a system implies the system itself does work on external systems. The work-energy theorem has been stated here in the “microscopic” view. In the microscopic view, heat and temperature do not exist. In the “macroscopic” viewpoint, heat and temperature exist and require additional comments regarding the relation between work and energy, but that is the subject of a later chapter.

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**Types of Energy**

There are many types of energy. The energy in the example of the rubber band is called potential energy. Potential energy of a system is caused to internal forces in that system. Potential energy is increased by doing work against the internal forces. The potential energy of a stretched state of a rubber band or spring is called elastic potential energy. Stretching against elastic forces increases the elastic potential energy. An approximate rule for elastic energy, that works well for most stretchable materials, is Hooke's law that says the force $F$ required to stretch is proportional to the length of the stretch.

$$F = kx$$

where $k$ is a constant that depends on the material, size and design of the object being stretched. The force increases with stretching. Calculus (a branch of mathematics) can be used to show that, for a material obeying Hooke's law, the change in elastic potential energy is $\frac{1}{2}kx^2$.

Gravitational potential energy is caused by separating two bodies that have gravitational attraction on each other. For example, the state of a body lying on the surface of the Earth can be taken as a reference state with zero gravitational potential energy. Raising the body to a height above the surface of the Earth requires force to separate the body from the Earth. A very important point is that this gravitational potential energy is not merely energy of the body raised, but is an energy of the system consisting of the body and the Earth. The body and the Earth share this potential energy. However, it is common for people (including engineers and scientists) to speak of this potential energy as being energy of the body alone (even though this is not correct). We know the weight $w$ (a force) of a body of mass $M$ on Earth, where the acceleration due to gravity is $g$, is $Mg$. This weight is a downward force. To lift the body to a height $h$, a force of at least the same magnitude must be applied upward. Thus, the
minimum work done to lift the body to height \( h \) is \( Mgh \) (where \( g \) is the magnitude of \( g \)). Thus, the gravitational potential energy \( V_{\text{grav}} \) of system consisting of the raised body and Earth is \( Mgh \) relative to the state with height zero.

\[
V_{\text{grav}} = Mgh
\]

Elastic potential energy and gravitational energy are two types of potential energy.

Another type of energy is kinetic energy. Kinetic energy is energy due to motion. The natural reference state, where we will take kinetic energy to be zero, is the state without motion. Consider a body at rest. This is our reference state for specifying the kinetic energy of the body. If the body is pushed with a net force \( F \) over a displacement \( x \) and both vectors \( F \) and \( x \) have the same direction, then the kinetic energy will increase to \( F \times x \) where \( F \) and \( x \) are the magnitudes of the force and displacement vectors. Calculus can be used to show that the kinetic energy \( K \), relative to the reference state where the body was at rest, is \( \frac{1}{2} m v^2 \) where \( v \) is the speed resulting from the work being done.

\[
K = \frac{1}{2} mv^2
\]

Remember that energy is a relative quantity because it is specified relative to a reference state. I chose the state of being at rest to be the reference state, but Galilean relativity shows that another observer can view the initial and final velocities to be different than I view them. The state I say has an object at rest may appear to another observer to be a state with the object in motion. Thus, an observer in one frame may measure the kinetic energy differently than an observer in another frame.

Potential energy \( V \) and kinetic energy \( K \) are known as mechanical energy. The total mechanical energy \( E_{\text{mech}} \) of a body or a system is the sum of all the potential and kinetic energy.

\[
E_{\text{mech}} = K + V
\]

Types of energy other than the mechanical energies include heat, chemical potential energy, energies of fields or radiation such as the electromagnetic energy of light, nuclear energy and mass energy. These energies are beyond the scope of this chapter.

**Conservation of Energy**

Since the change in energy of a closed system is equal to the total work done on the system, if a closed system has zero total work done on it, then the change in energy is zero. Thus, a very important physical law, called the “Law of Conservation of Energy” follows.

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**Law of Conservation of Energy:** If the total work done on a closed system is zero, then the energy of that system does not change.

Most books state this law in other ways. A common alternative is “energy cannot be created nor destroyed.” This law allows one form of energy to be transformed into another form of energy. Thus, people often add to this law the statement that “energy can be transformed from one form to another.”

Here are some examples of energy being converted from one form to another.

- A rubber band is stretched by hand to a length $x$ to increase its potential energy to $\frac{1}{2}kx^2$ and then the rubber band is released so it shoots out away from the hand or hands that stretched it. The potential energy is converted to kinetic energy. Since the energy cannot be destroyed, the kinetic energy would be the same as the $\frac{1}{2}kx^2$ potential energy was except that some of that potential energy may have been converted to heat.

- Consider a body with mass $M$. Work can be done on the system consisting of the body and the Earth, and the work separates the body and the Earth by raising the body to a height $h$. The work required to lift the body is $Mgh$. Thus, the potential energy of the system, after lifting the body, is $Mgh$. If the body is then dropped from height $h$, the potential energy is lowering as it falls but the kinetic energy is rising. When the body gets back down to its original position, it has kinetic energy equal to the potential energy $Mgh$ that it had at height $h$. When the body strikes the Earth and stops, its kinetic energy is converted to heat energy.

- A floodlight converts electrical energy to light and light has electromagnetic energy.

- A chemical bomb explodes converting chemical potential energy to kinetic energy. The physics students detonating the bomb are killed. This is very sad because they don't learn physics anymore.

**Linear Momentum and Energy of a System**

Earlier in this chapter, we learned that if the net external force on a closed system is zero, then the total linear momentum of the system is conserved. In this chapter we also learn that if the total work done on a closed system is zero, then the energy of the system is conserved. Thus, if we have a closed and isolated system, both linear momentum and energy are conserved. When parts of such a system interact with each other (through internal forces), linear momentum and energy can be transferred between the parts while conserving the total linear momentum and total energy of the system. The following examples illustrate the usefulness of these conservation concepts and important additional concepts.
• Collisions are called “perfectly elastic” collisions if kinetic energy is conserved in the collision. Some books may say the collision is perfectly elastic if the total mechanical energy is conserved in the collision. Suppose two bodies have the same mass, the first body is moving, the second body is at rest and then the first body collides with the second. If the collision is perfectly elastic, then both the linear momentum and the kinetic energy is conserved in the collision. The only way both quantities can be conserved is if the first body comes to rest in the collisions and the second body moves with the same velocity after the collision as the first had before collision. In an inelastic collision, the kinetic energy is not conserved and there are other possibilities. For example, in an inelastic collision the first body may stick to the second and both move together.

• A playground teeter-totter is a type of lever and fulcrum made for recreation. Two children sit on opposite sides of the device. When one child lowers, the other rises. Neglecting generation of heat due to friction at the center fulcrum, the mechanical energy should be conserved. Thus, as the potential energy of one child and Earth lowers, the potential energy of the other child and Earth should rise. If the device is used properly with proper placement (or balance) of the children on the device, the children can easily operate the device without significant kinetic energy and without the need to exert much force. The two children can easily move slowly up and down. As the potential energy on one side is rising, the potential energy on the other side is lowering by the same amount so the total potential energy is not changing. This is easy to see if the two children have the same mass and they sit the same distance from the center fulcrum (but on opposite sides). If one child is heavier, the heavier child must sit closer to the center fulcrum to keep the proper balance. Without proper balance, one child needs to push hard to operate the device. The total potential energy still does not change because changes in height are smaller for the heavier child.

• A ball rolling forward and back on a U-shaped ramp gains kinetic energy when rolling down but loses potential energy. When the ball rolls up, the kinetic energy decreases as the potential energy increases. Neglecting generation of heat by friction, the mechanical energy is conserved. Thus the potential and kinetic energies are merely being converted into each other.

• A common carnival device, used by entertainers and con artists, consists of a heavy ball (like a bowling ball) and a track on which the ball is to roll. The track has hills and valleys, and the end is higher than the rest of the track. The start of the track is the lowest part of the track. The track is made so the forces of friction between the ball and the track are small. This device is used in a game in which the ball is to be rolled from the start. As the ball rolls up and down the hills of the track, some kinetic energy is converted to potential energy as the ball rolls up any of the
hills of the track, but the potential energy is converted back to kinetic energy as it rolls down the hills. If a ball is rolled from the start but lacks enough kinetic energy to clear the first hill, the ball rolls back to the start. Even if the ball has enough kinetic energy to get over the first hill, it eventually will get to a part of the track where all the kinetic energy has been converted to potential energy and the ball will begin to roll back. The challenge given to the user of such a device is to roll the ball in such a way that the ball does not return to the start but remains in one of the valleys. I sometimes assign a homework problem that deals with this clever device. The main question of the homework problem is this: “Since the frictional forces are small, is the player rolling the ball likely to get the ball to stay in one of the valleys?”

**Power**

Power is the rate at which energy is converted or transferred. Power is measured in watts which are often denoted by a capital W.

\[ W = \frac{J}{s} = \frac{N \cdot m}{s} = \frac{kg \cdot m^2}{s^3} \]

Chemical batteries that power modern electronics produce electrical energy by electrochemical reactions. If an electrochemical reaction converts chemical potential energy to electrical energy at a rate of 200 joules in 10 seconds, the average power provided by the reaction in those 10 seconds is

\[ \frac{[200 \ J]}{[10 \ s]} = 20 \ J/s = 20 \ W \]

**Mechanical Energy and Efficiency**

Often some energy seems to be missing when energy is converted from one form to another because some of the energy was converted to heat. Engineers speak of energy being “lost” as heat. Heat is a form of energy that results from friction, chemical reactions, electrical processes. When you notice a mechanical system, a chemical system or an electrical circuit getting hot, that is because some energy was converted to heat.

Mechanical engineers like to design efficient machines. Machines are mechanical devices that do work. The machines are considered efficient if they handle energy without generating too much heat. Machines require energy input to make them work, and they do work. A consequence of the Law of Conservation of Energy is that the amount of work the machines do must be less than or equal to the
energy input. The designer of a machine tries to get the work done by the machine to be close to the energy input. The efficiency is a measure of success.

\[
\text{Efficiency} = \frac{\text{[work done]}}{\text{[energy input]}}
\]

Efficiency can be expressed as a fraction, decimal value or percent. A common reason the efficiency is not 100% is that some energy is “lost” as heat. Since many machines provide work over an interval of time or continuously, we can also calculate efficiency in terms of power.

\[
\text{Efficiency} = \frac{\text{[rate at which work is done]}}{\text{[rate at which energy is input]}}
= \frac{\text{[power provided as a work rate by the machine]}}{\text{[power input]}}
\]

Thus, a machine that does work at a rate of 30 W with an input power of 40 W has an efficiency of 75%.

There are types of machine that have theoretical upper limits on efficiency. Heat engines have upper limits. Thus, car manufacturers consider replacing internal combustion with electric motors because they cannot make internal combustion engines with efficiencies greater than a certain limit. Internal combustion loses more than 30% of its input energy as heat.

**Example Calculations**

Some example calculations were done in previous sections of this document. In this section, some more examples are provided.

**Example 01.** Suppose a body pushes on a second body with a force \( \mathbf{F}_{12} \) of 10 newtons East. We can use Newton's Third Law of Motion to calculate the force \( \mathbf{F}_{21} \) the second body exerts on the first body.

\[
\mathbf{F}_{21} = - \mathbf{F}_{12} = - 10 \text{ N East} = 10 \text{ N West}
\]

Thus, the second body pushes with a force of 10 newtons West on the first body.

**Example 02.** Suppose a body has mass \( m = 2.0 \text{ kg} \) and velocity \( \mathbf{v} = 3.0 \text{ m/s} \) to the right. Then its linear momentum \( \mathbf{p} \) can be calculated.

\[
\mathbf{p} = m\mathbf{v} = 2.0 \text{ kg} \times 3.0 \text{ m/s} \text{ to the right} = 6.0 \text{ kg m/s to the right}
\]
Notice the directions of the linear momentum of a body and the velocity of the body are always the same.

**Example 03.** Suppose the linear momentum \( p \) of a body is changing at a constant rate \( \Delta p/\Delta t \) of 20 kg m/s\(^2\) to the right. We can calculate the net force \( F_{net} \) on this body. Notice the units are metric units so the value of the constant \( K \) is one in the formula

\[
F_{net} = K \frac{\Delta p}{\Delta t} = \frac{\Delta p}{\Delta t} = 20 \text{ kg m/s}^2 \text{ to the right} = 20 \text{ N to the right}
\]

The net force is 20 newtons to the right. (The constant \( K \) would not have been 1 if we had mixed units such as wanting the force expressed in pounds of force rather than the metric newtons.)

**Example 04.** Suppose the net force on a body \( F_{net} \) is 2 newtons upward for a time \( \Delta t = 2 \text{ seconds} \). We can calculate the impulse \( I_m \) and the change in linear momentum \( \Delta p \). Since metric units are used, the value of the constant \( K \) is one in the formula

\[
I_m = F_{net} \Delta t = K \Delta p = \Delta p
\]

\[
\Delta p = F_{net} \Delta t = 2 \text{ N x 2 s upward} = 4 \text{ N s upward} = 4 \text{ kg m/s}^2 \text{ s upward} = 4 \text{ kg m/s upward}
\]

Thus, the impulse is 4 N s upward and the change in linear momentum is kg m/s upward.

**Example 05.** Suppose a force \( F \) is exerted on a body and the body moves a displacement \( x \). The amount of work \( W \) done on the body depends on the directions of \( F \) and \( x \). Suppose the magnitude \( F \) of force \( F \) is 3 N and the magnitude \( x \) of displacement \( x \) is 2 m. If the force and displacement are in the same direction, then the work done on the body is

\[
W = Fx = 3 \text{ N x 2 m} = 6 \text{ J}
\]

If the force and displacement are in opposite directions, the work done is

\[
W = -Fx = -6 \text{ J}
\]

If the force is perpendicular to the displacement, then the work done is zero.

**Example 06.** Suppose the speed \( v \) of a body is 5.0 m/s and the body has a mass \( m = 10 \text{ kg} \). We can calculate the kinetic energy \( K \) of the body.

\[
K = \frac{1}{2} mv^2 = \frac{1}{2} \times 10 \text{ kg} \times (5.0 \text{ m/s})^2 = 125 \text{ kg m}^2/\text{s}^2 = 125 \text{ J}
\]
If the body is brought to rest, the kinetic energy must be converted to some other form of energy because energy is conserved. For example, if the direction of the body's velocity $v$ is upward, the gravity of the Earth can bring the body to rest. Then the kinetic energy of the body is converted to gravitational potential energy of the Earth and body. Notice that the gravitational potential energy belongs to the system of the Earth and body rather than merely to either the body or the Earth. When gravity brings the body to rest, the potential energy is 125 J because that was the value of the kinetic energy.